Lecture 1

Why fatigue is so important?

Fatigue Design Overview

800 ANNI 1222·2022

UNIVERSITÀ DEGLI STUDI DI PADOVA

University of Padua (Italy)

The university is conventionally said to have been founded in 1222 (which corresponds to the first time when the University is cited in a historical document as pre-existing, therefore it is quite certainly older)[citation needed] when a large group of students and professors left the University of Bologna in search of more academic freedom ('Libertas scholastica')

15 February 1564 – 8 January 1642

The University of Padova was established in 1222, after a group of students and teachers decided to come here from Bologna.

NTNU

45000
students

7500
person-years

400
doctoral degrees

NOK 9,4 billion
annual budget

https://www.ntnu.no/gloshaugen

NTNU

https://www.trondheim.kommune.no/

NTNU

https://www.ntnu.no/imt https://www.ntnu.edu/mtp/laboratories/nanotestlab

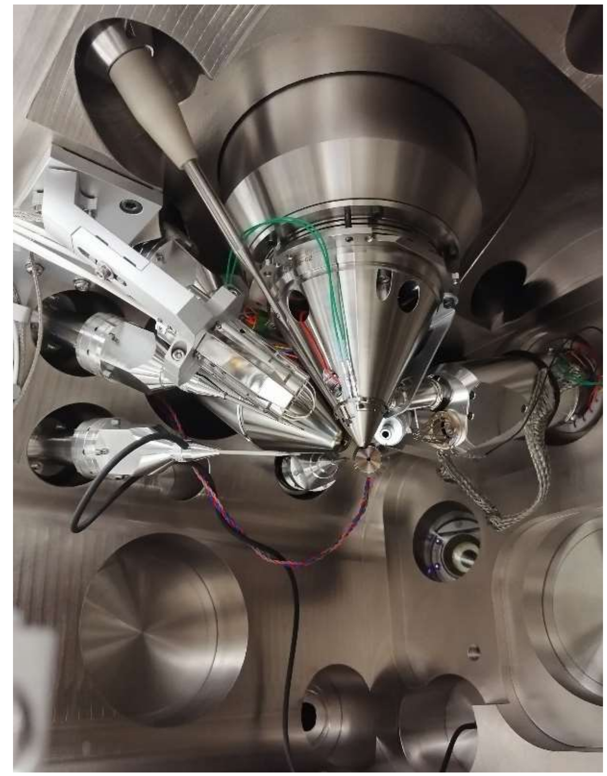
NTNU

Dual-beam platform microscopy: FIB-SEM

Focused Ion Beam-Scanning Electron Microscope

FEI Helios NanoLab DualBeam FIB

Commonly used for:
- TEM sample preparation
- Computer chip repair
- Circuit modification
- 3D FIB *tomography*



NTNU

FEI
Titan G2 60-300

JEOL
2100F

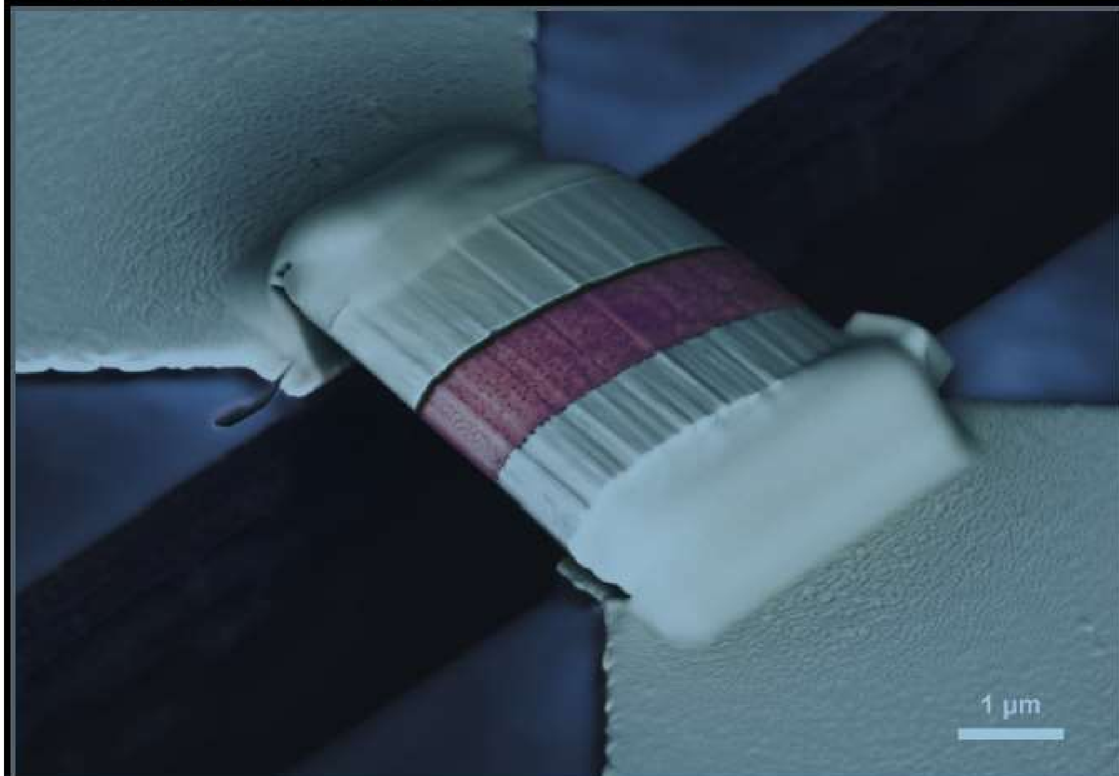
JEOL
ARM200F

JEOL
2100F

JEOL
2100

NTNU

Markus Lid



Electron Crossing - This is a colorized SEM image taken with FIB Helios G4 during the fall of 2020. It is imaged a thin film metal-insulator-semiconductor capacitor that was cut using Ga ion beam (also FIG G4), lifted out and placed on a separate chip where it was connected to the electrodes by Pt deposition. It is placed over a 3um wide trench which is necessary for experiments. The project which is led by Fritz Prinz at Stanford University, is studying properties of exited electronic states in semiconductor structures.

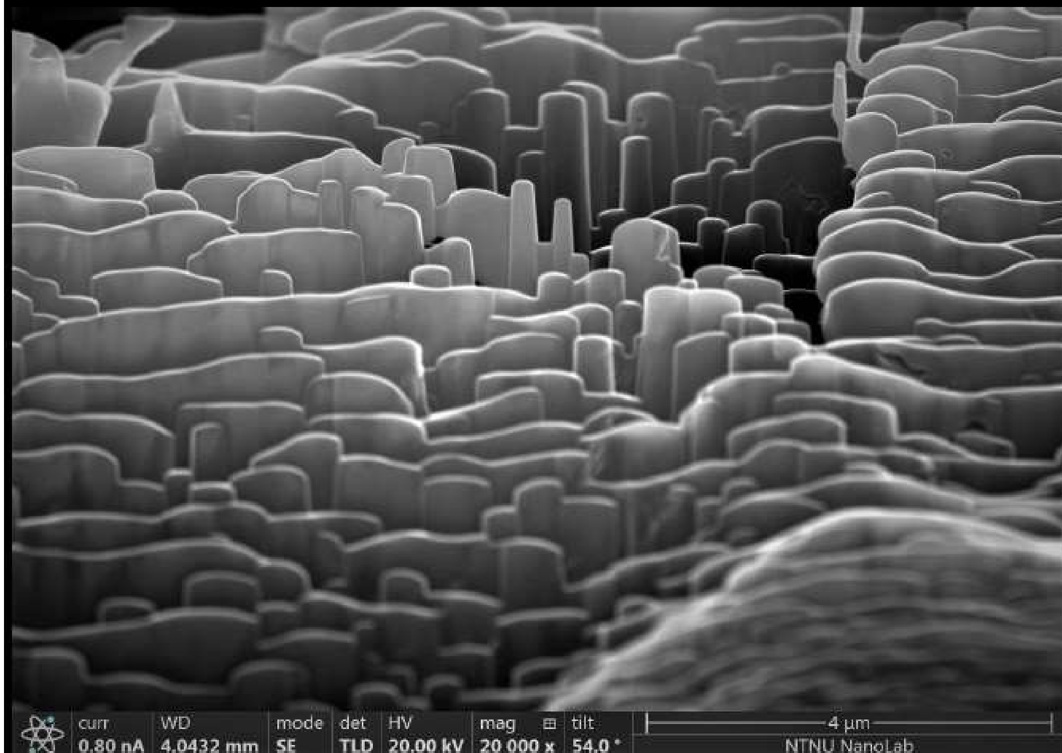
NTNU

Erik Roede

Fe3Sn2 magnetic lamella - 300 nm thickness. SE micrograph from the G4 with overlaid MFM two-pass phase image of the magnetic domain structure. Prepared by plan view FIB liftout and attached to biasing chip made at NanoLab with the MLA for in-situ studies in AFM. The goal of this project is to study the influence of geometry, field and currents on the magnetic texture in the frustrated ferromagnet Fe3Sn2, ultimately aiming for stabilizing and controlling magnetic skyrmions for nanoelectronic device applications.

NTNU

Ambra Celotto



Flexible graphite – also known as expanded graphite or exfoliated graphite – is produced from purified natural graphite flakes. The foils obtained by the compression of those result to be composed by several thin layers in a multiscale structure. By FIB slicing this material in a tilted direction, I found myself in front of a tiny (and highly populated) city under the moonlight.

https://www.ntnu.no/imt https://www.ntnu.edu/mtp/laboratories/nanotestlab

NTNU

Shock Tube Facility

18,2 m long tube divided into six sections. The tube ends in a 5.1 m dump tank. Threaded holes in the tube floor enable mounting of test specimens in the test section. Windows in the test section and the dump tank allow high-speed cameras to investigate the structural response during an experiment.

Pendulum Impactor

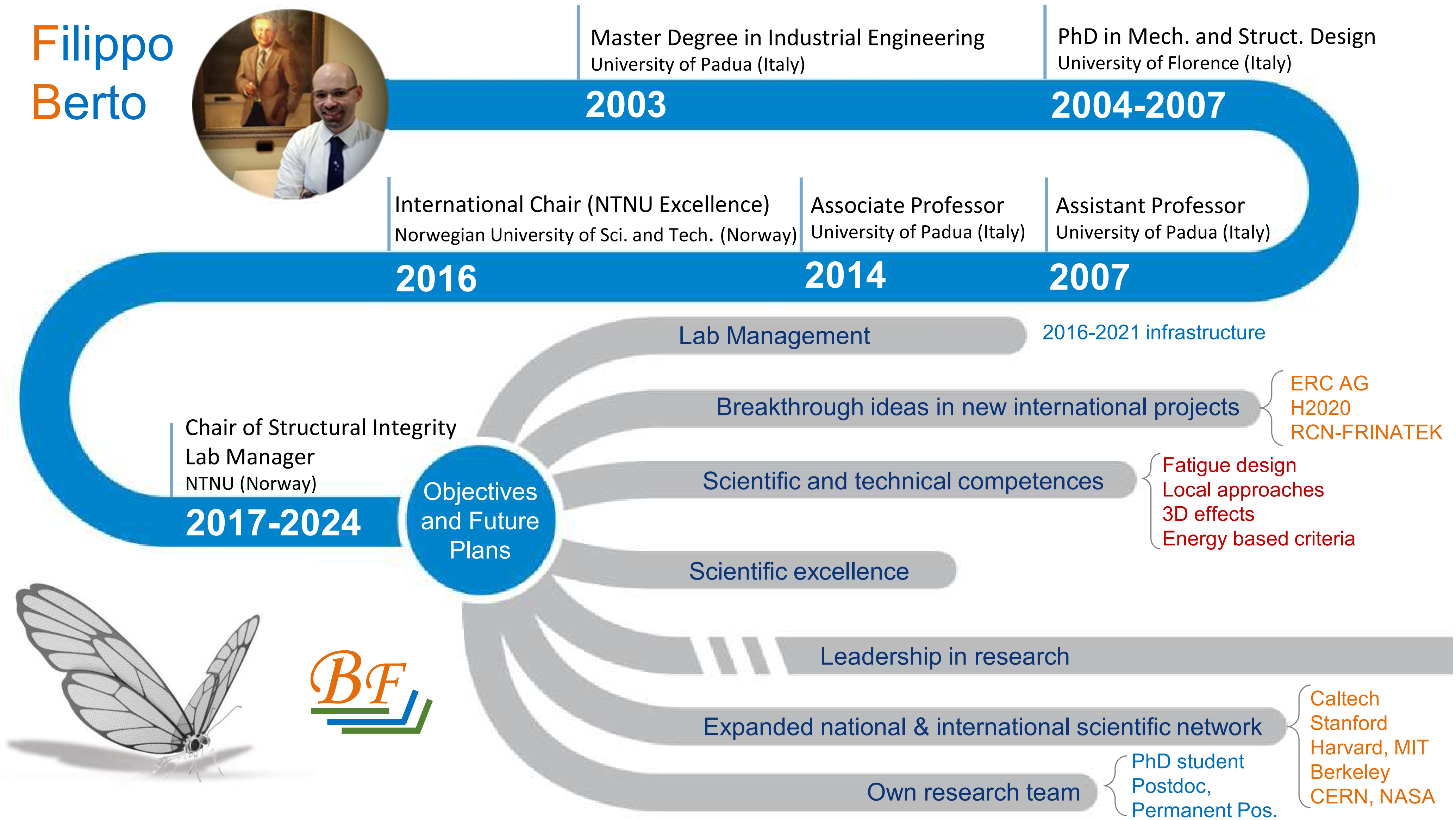
The pendulum accelerator, is a device for impact testing of components and structures. The test rig accelerates a trolley on rails towards a test specimen fixed to a reaction wall. The accelerating system consists of an arm that is connected to a hydraulic/pneumatic actuator system. The maximum energy delivered to the trolley is aproximately 500 kJ.

Gas Gun

A compressed gas gun for ballistic impact studies. A variety of projectile geometries can be fired, with a maximum velocity of 1000 m/s.

Split-Hopkinson Tension Bar

A device for material testing at strain rates in the range between 100 and 1500 s–1. Data is recorded with strain gauges and high-speed cameras. An induction heater facilitates tests at elevated temperatures.


Filippo
Berto
Master Degree in Industrial Engineering
University of Padua (Italy)
2003
PhD in Mech. and Struct. Design
University of Florence (Italy)
2004-2007
International Chair (NTNU Excellence)
Norwegian University of Sci. and Tech. (Norway)
2016
Associate Professor
University of Padua (Italy)
2014
Assistant Professor
University of Padua (Italy)
2007
Chair of Structural Integrity
Lab Manager
NTNU (Norway)
2017-2024
Objectives and Future Plans
Lab Management
2016-2021 infrastructure
Breakthrough ideas in new international projects
ERC AG
H2020
RCN-FRINATEK
Scientific and technical competences
Fatigue design
Local approaches
3D effects
Energy based criteria
Scientific excellence
Leadership in research
Expanded national & international scientific network
Caltech
Stanford
Harvard, MIT
Berkeley
CERN, NASA
Own research team
PhD student
Postdoc,
Permanent Pos.
BF

NTNU

Research topics

Advanced Design Criteria for Fatigue Assessment
Degradation of Materials and Structures
Interaction between Mechanical Performances and Manufacturing Processes
Metamaterials






People

Ass. Prof. Chiara Bertolin (Onsa
Fellowship)
Ass. Prof. Nicola Paltrinieri (Onsager Fellowship)
Ass. Prof. Javad Razavi
Ass. Prof. Chao Gao
Ass. Prof. Jan Torgersen
Adjunct Prof. Oystein Grong

1 Ass. Prof. 20% (CERN) 3 PostDocs
20 PhDs
50 master students/year
10 visiting researchers/year

https://www.ntnu.edu/mtp/research/dam

NTNU Fatigue Lab

Fatigue laboratory

Investments 2017-2019 3.5 MEuro

Axial Capacity from 500 N to 500 kN

Torque Capacity 10 Nm to 4000 Nm

Temperature from -100 C to 2000 C

Relative Humidity Control

Infrared Camera

Acoustic Emission

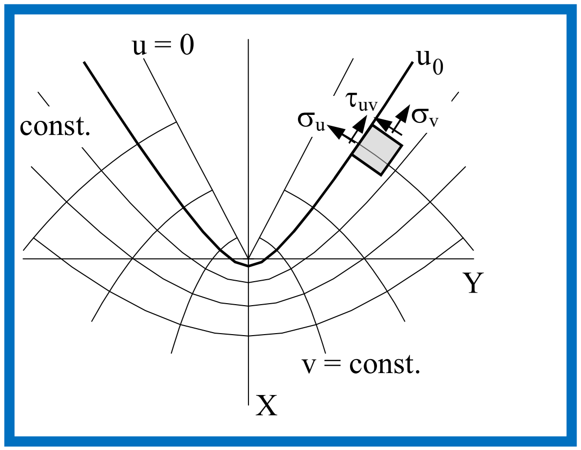
Electro-drop voltage

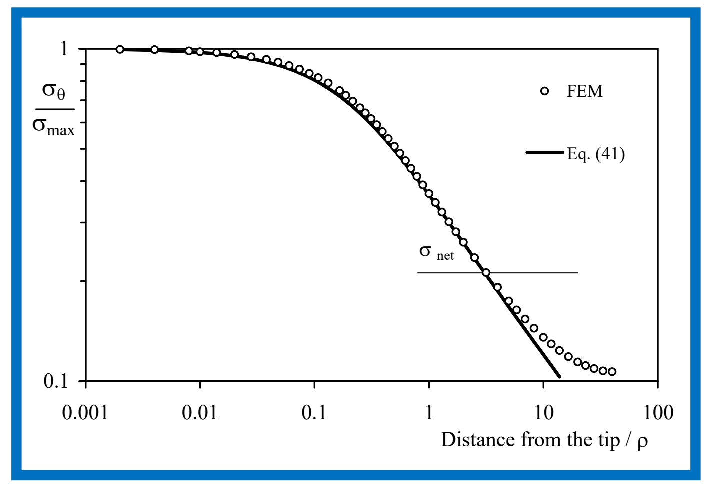
High speed camera

Flexible set-up for components testing

https://www.ntnu.edu/mtp/laboratories/mechtestlab

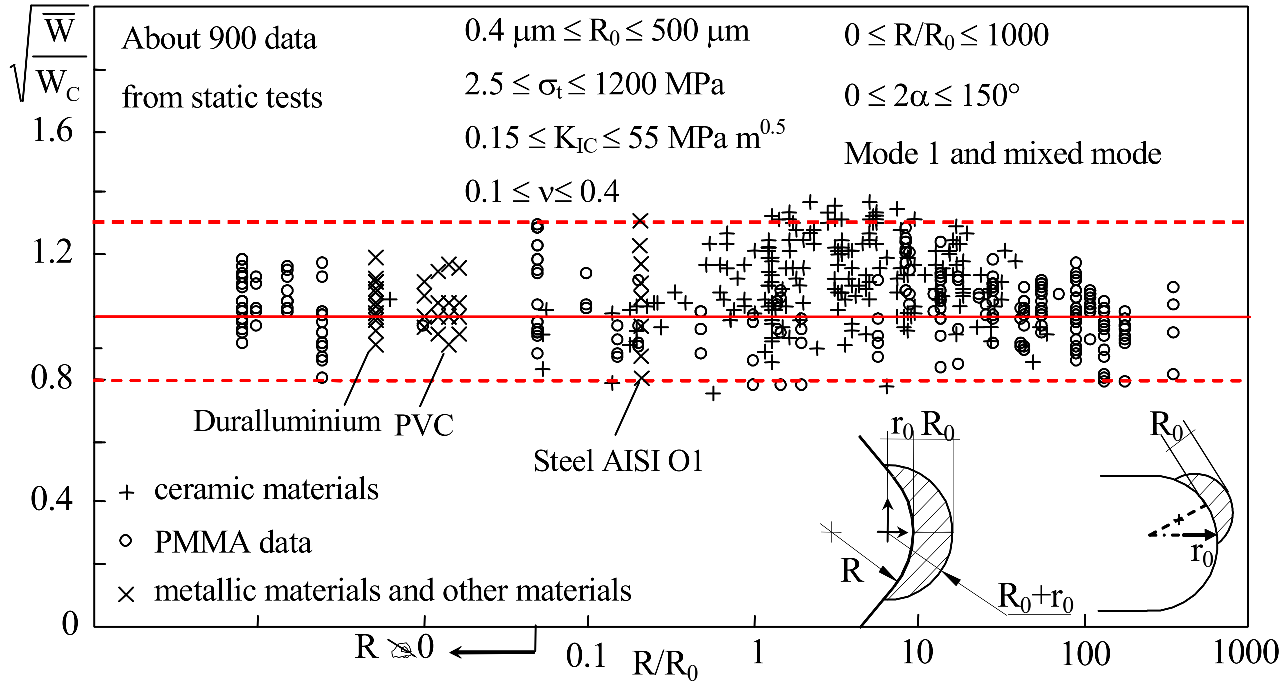
The Initial Path of My Research





$$(\sigma_\theta)_{\theta=0} = \frac{\partial^2 \Phi}{\partial x^2} = C_1(1+C_2)\frac{t+q}{q}\left[\frac{t}{q}x^{\frac{t}{q}-1} - \frac{1-q}{q}x_0^{\frac{t+q-1}{q}}x^{\frac{1-2q}{q}}\right]$$

Local Approaches in Fatigue



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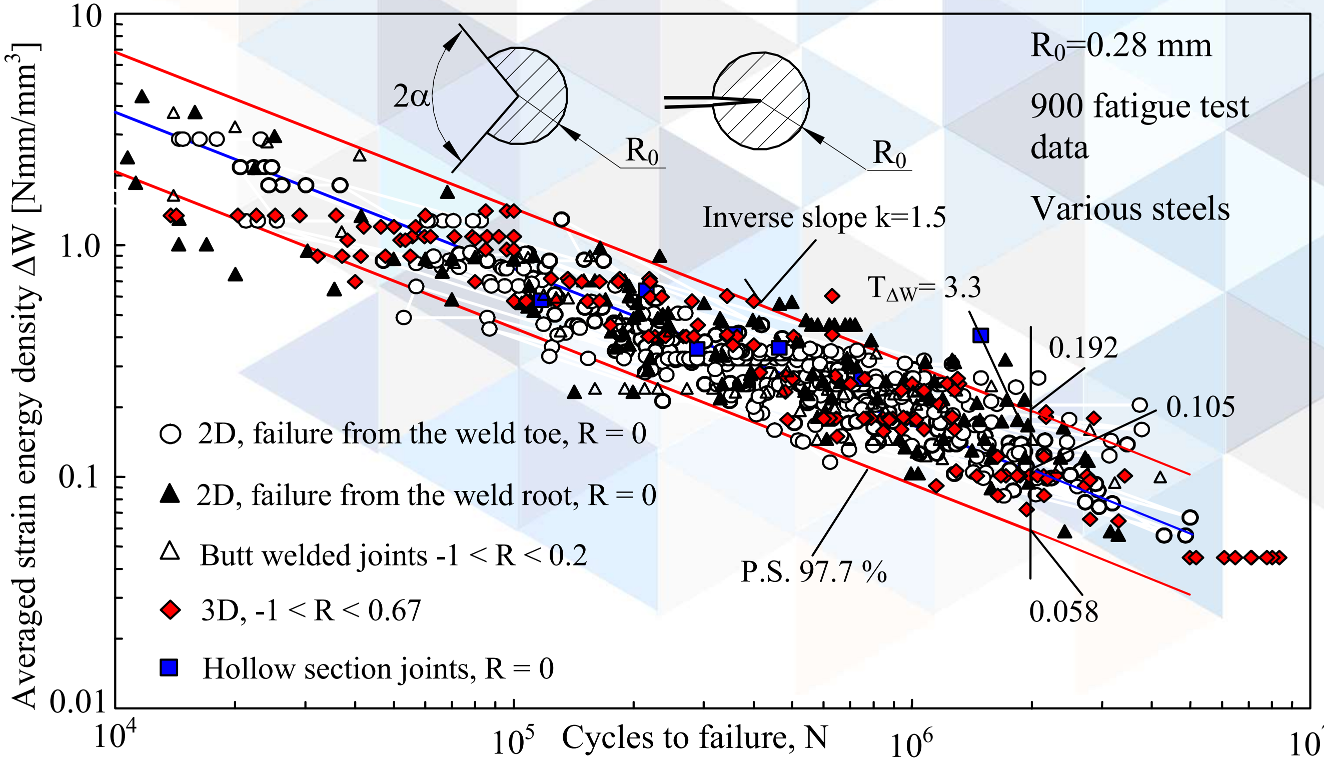
Local Approaches in Fatigue



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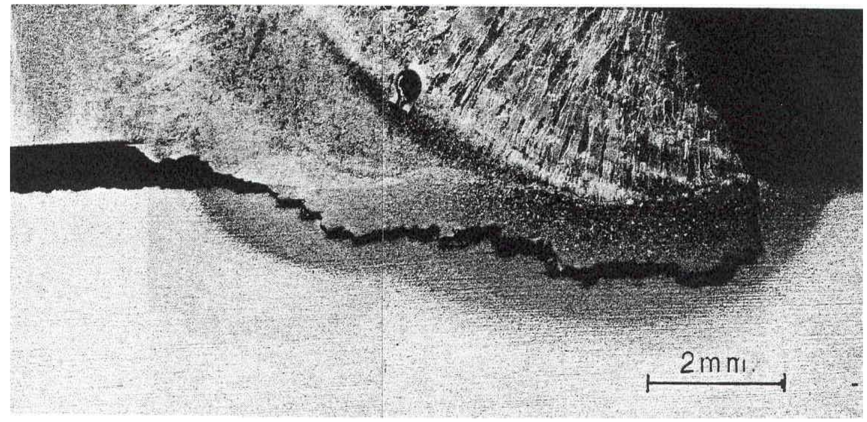
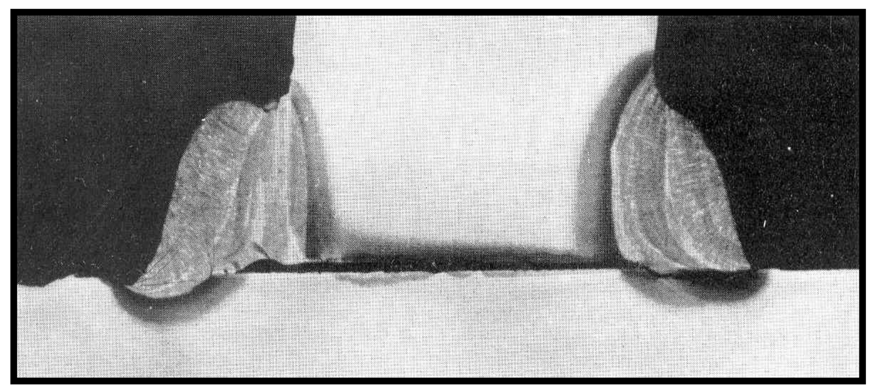
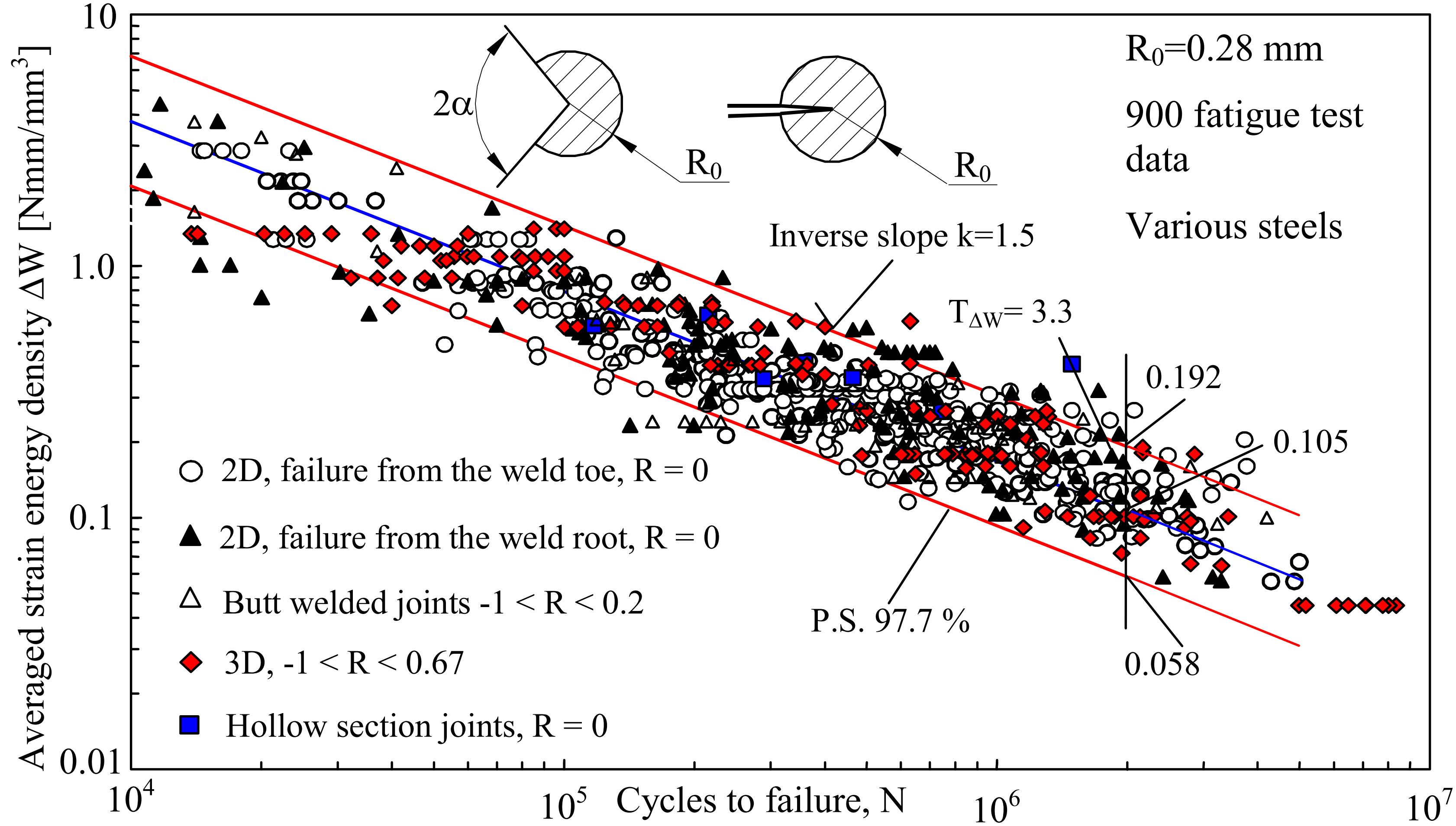
Fatigue of Welded Joints

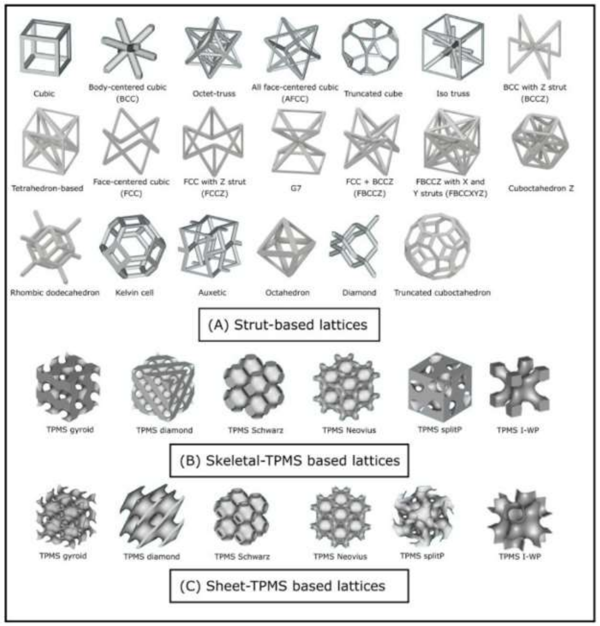




Fatigue of Welded Joints



Additive Manufacturing



Architected cellular materials: A review on their mechanical properties towards fatigue-tolerant design and fabrication

M. Benedetti, A. duPlessis,R.O. Ritchie, M. Dallago, S.M.J.Razavi, F. Berto

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Additive Manufacturing

Metal additive manufacturing in aerospace: a review.
Materials & Design, p.110008. Blakey-Milner, B., Gradl, P., Snedden, G., Brooks, M., Pitot, J., Lopez, E., Leary, M., Berto, F. and du Plessis, A., 2021.

Materials & Design

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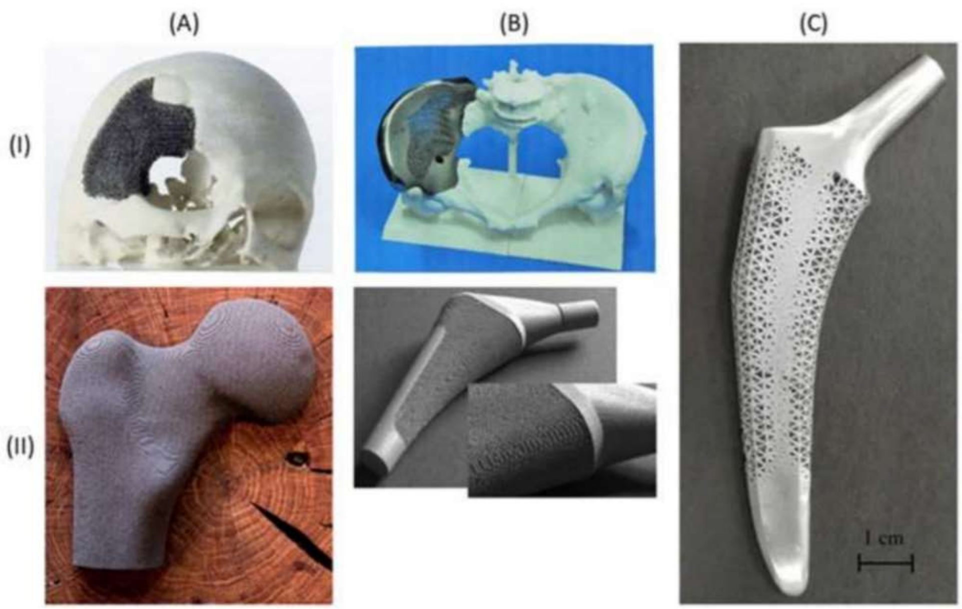
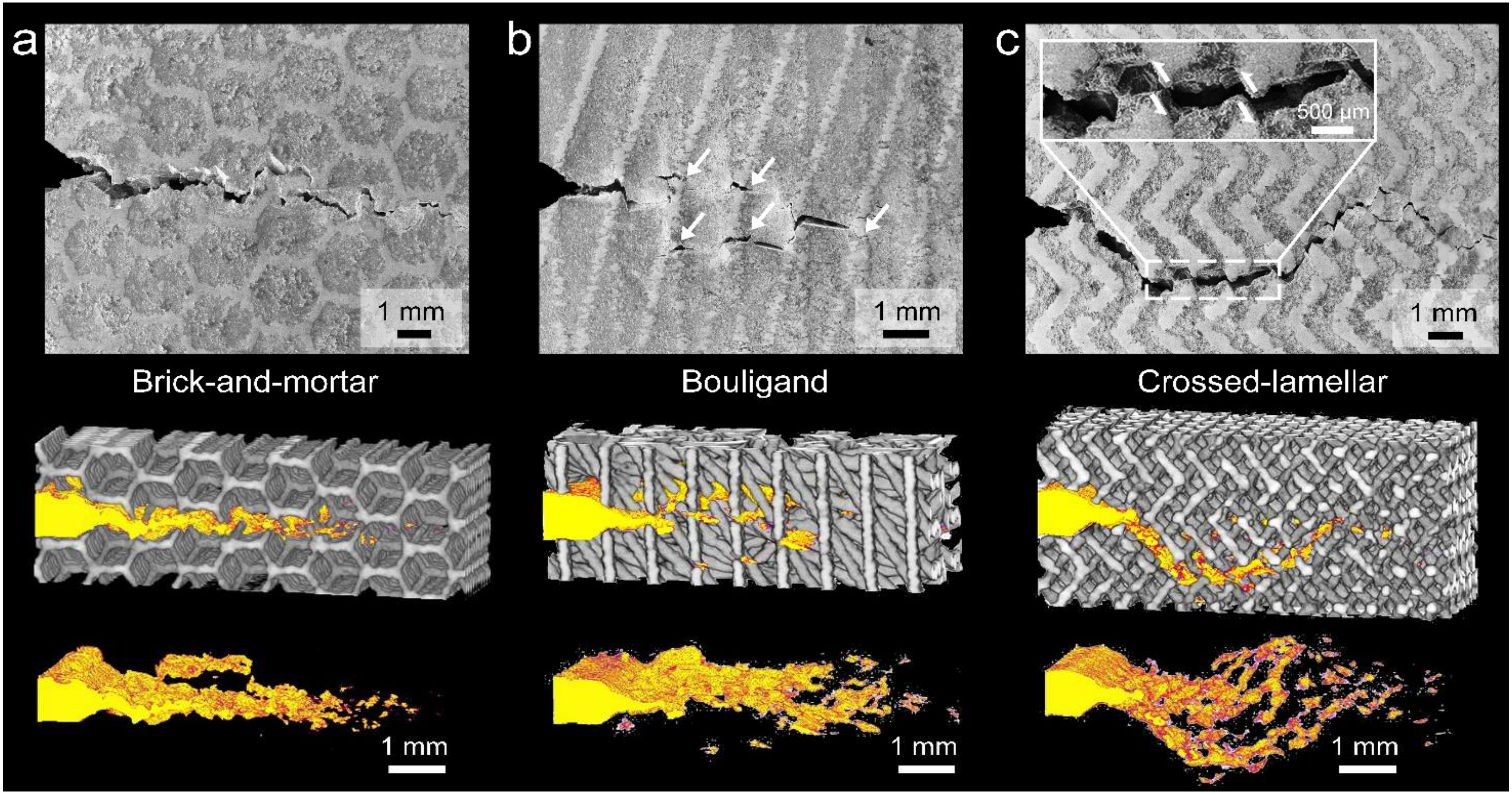
Additive Manufacturing



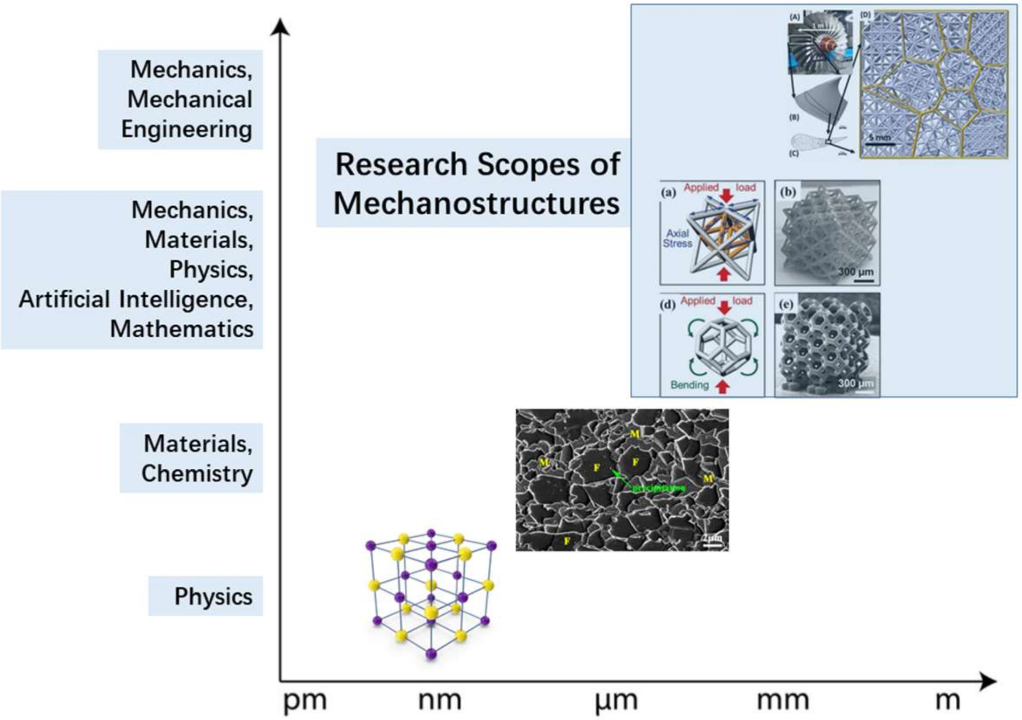
Properties and applications of additively manufactured metallic cellular materials: a review

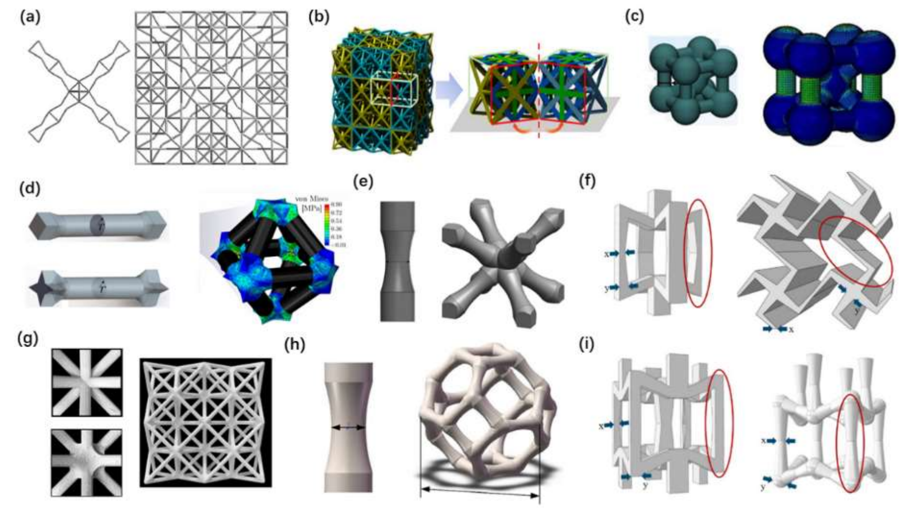
Additive Manufacturing

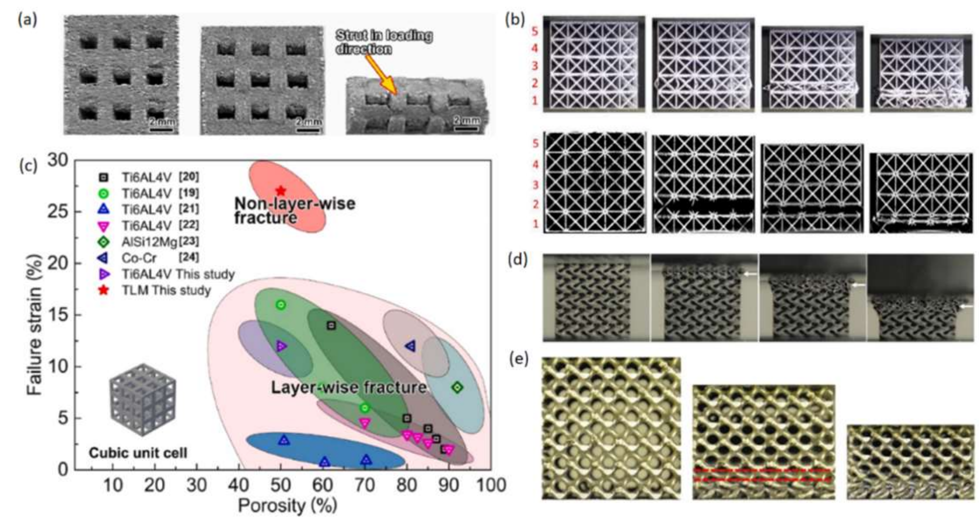


nature > nature communications

Additive Manufacturing







Progress in Materials Science

Supports *open access*

61.7 CiteScore

39.580 Impact Factor

National Infrastructure

Fatigue Lab 6 MEeuro

Smart-H 10 MEuro

HydroCen 19 MEuro

SAPIENZA
UNIVERSITÀ DI ROMA








RESERVED


AWARD OF MERIT


LEHIGH
hanics 2011
th-8th July 2011

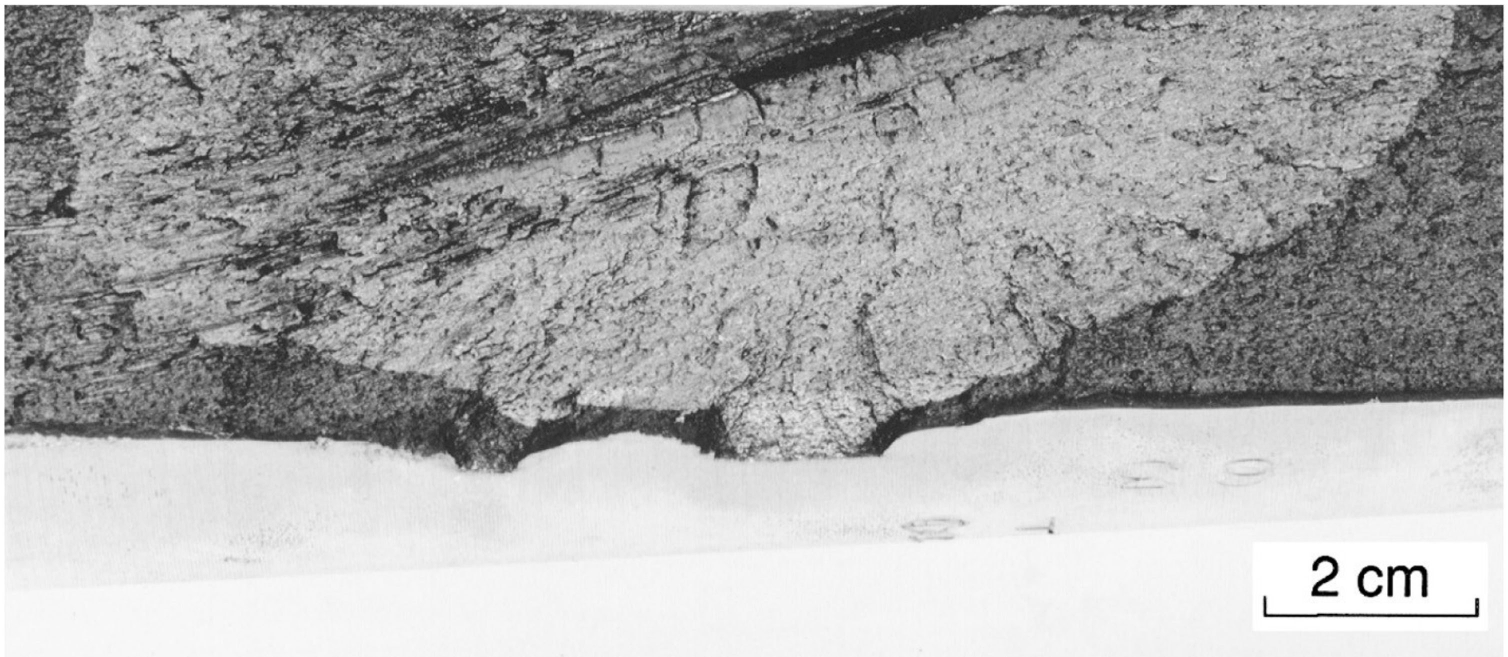
Lecture 1

Why fatigue is so important?

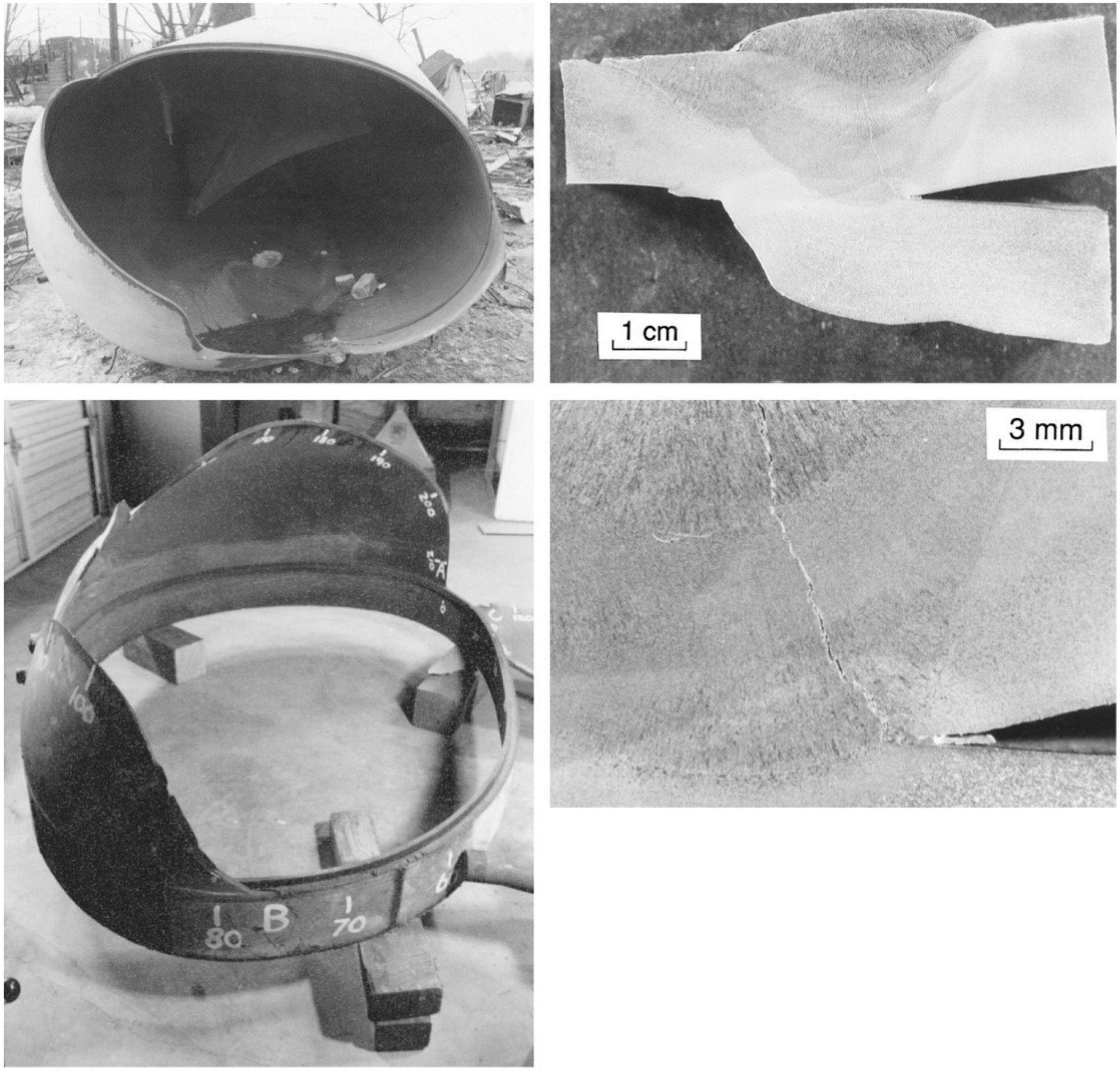
Fatigue Design Overview

Brittle fracture in an oil tanker

Fracture in a welded joint



Fatigue in a welded joint

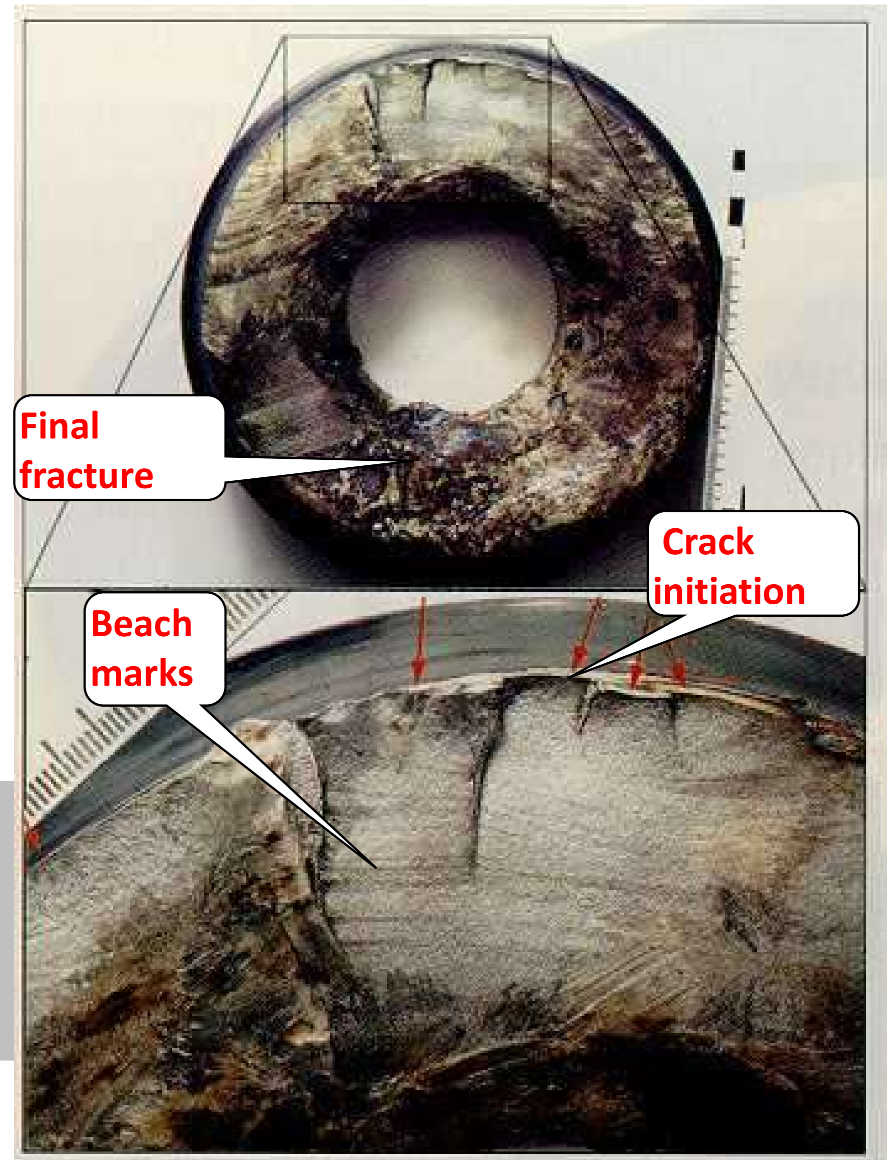


Failure in mechnical components

NSB train axle failures

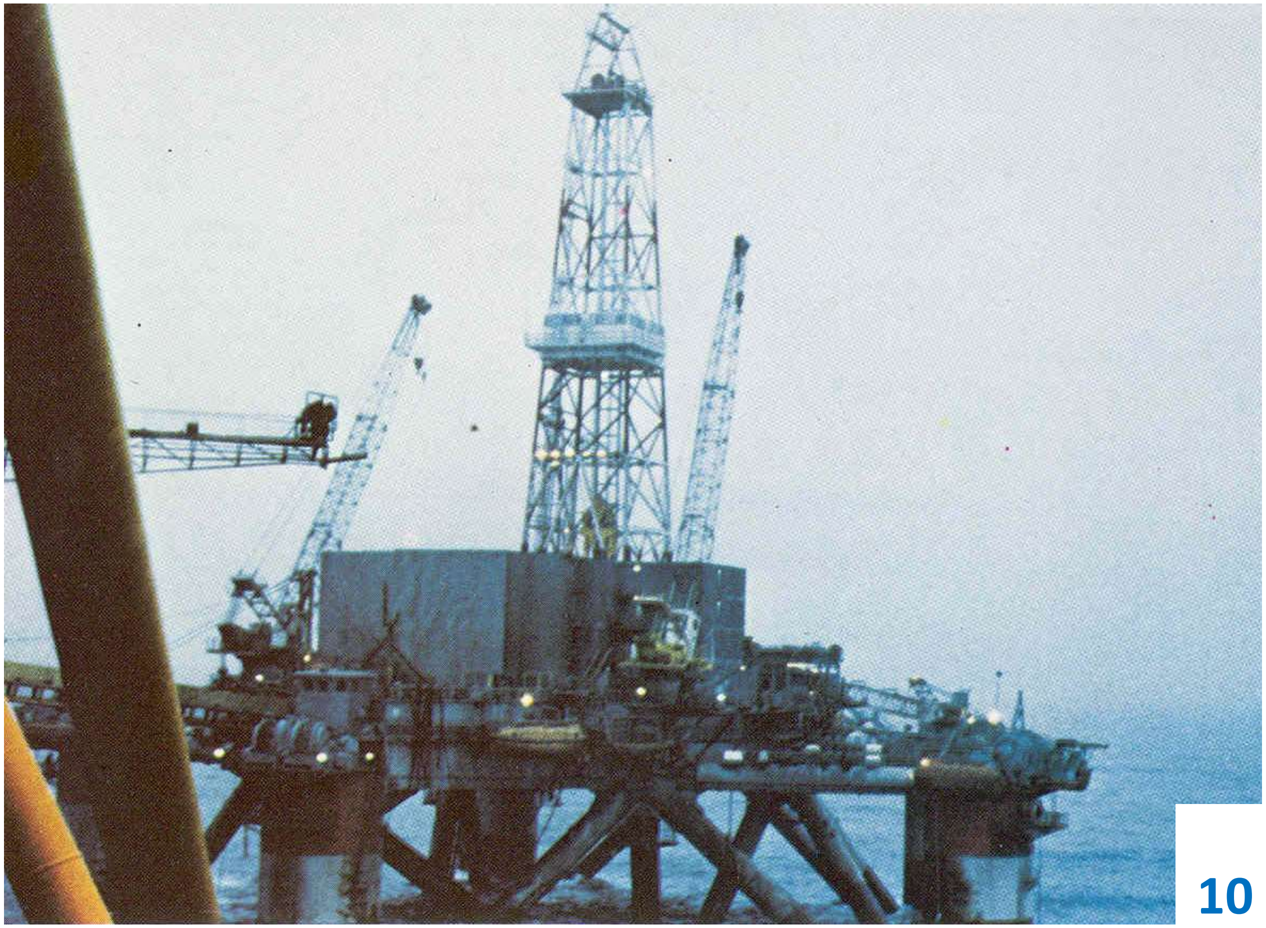
NSB Signature Train axle fractures

Summer 2002



Example **Failure of a welded structure**

The Alexander L. Kielland accident



Place: **Ekofisk field**

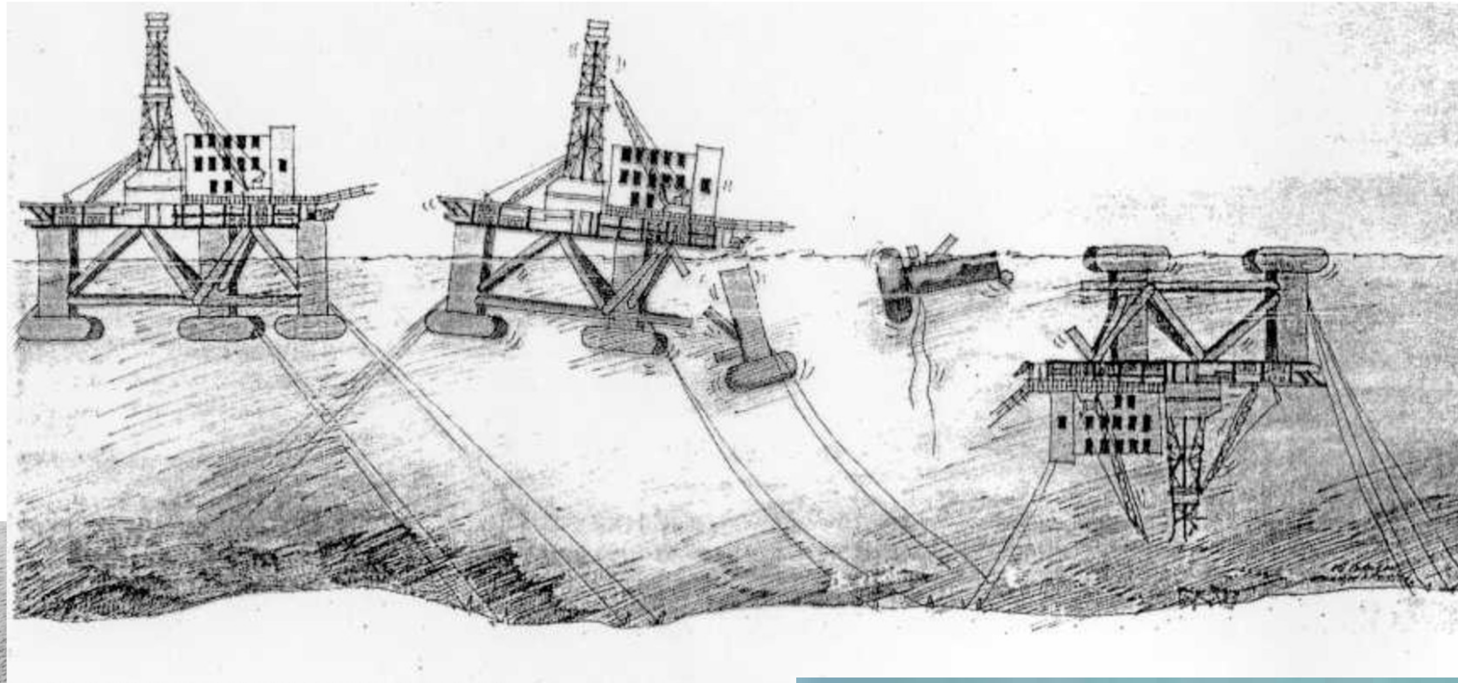
Time: **27 March, 1980, 18.30 hrs**

Persons killed: **123**

Survivors: **89**

10 similar platforms built
ALK platform delivered in 1976
Time from first failure in brace D6 to capsizing: 15 to 20 min

Example: The Alexander L. Kielland accident



Fatigue fracture surfaces

Three characteristic features of fatigue fractures:

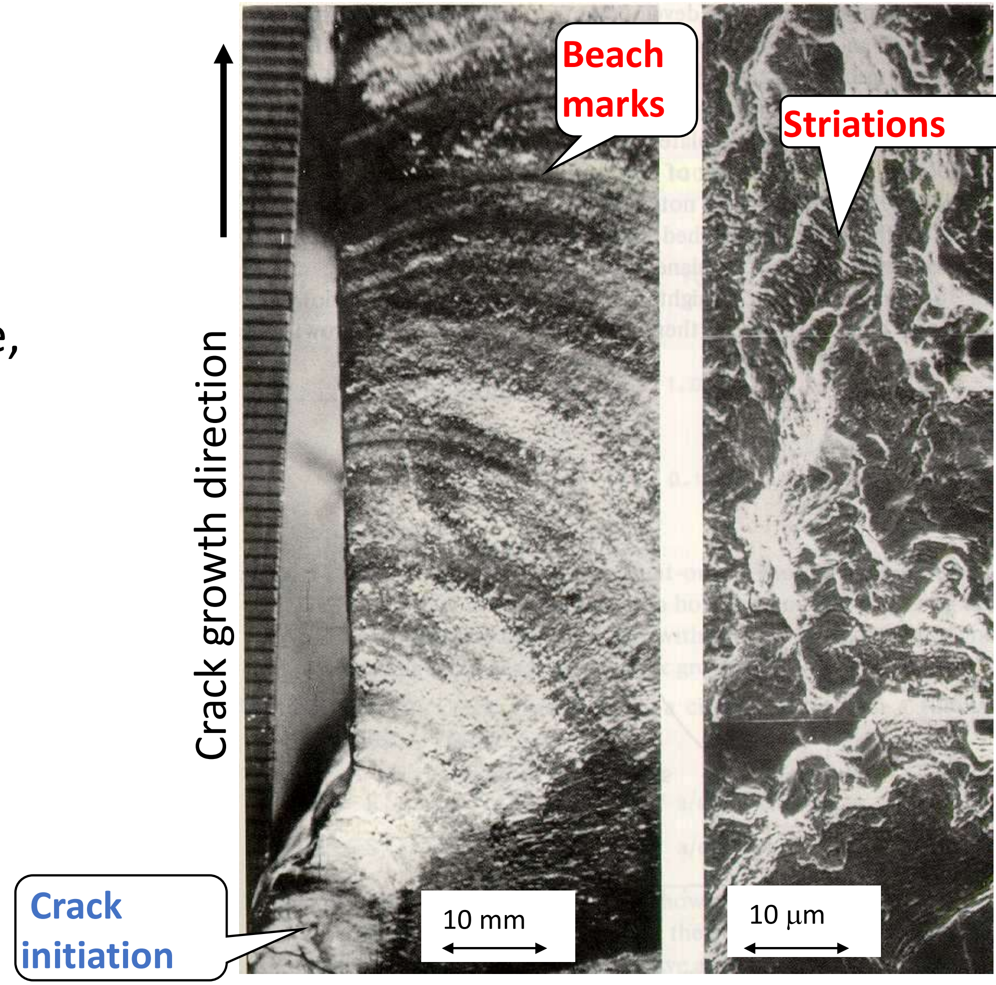
1. **Initiation point or points**
2. **Crack growth area**
3. **Final fracture**

Beach marks are lines visible to the naked eye, indicating changes in loading or corrosion conditions.

Striations indicate start-stop positions of the crack tip.

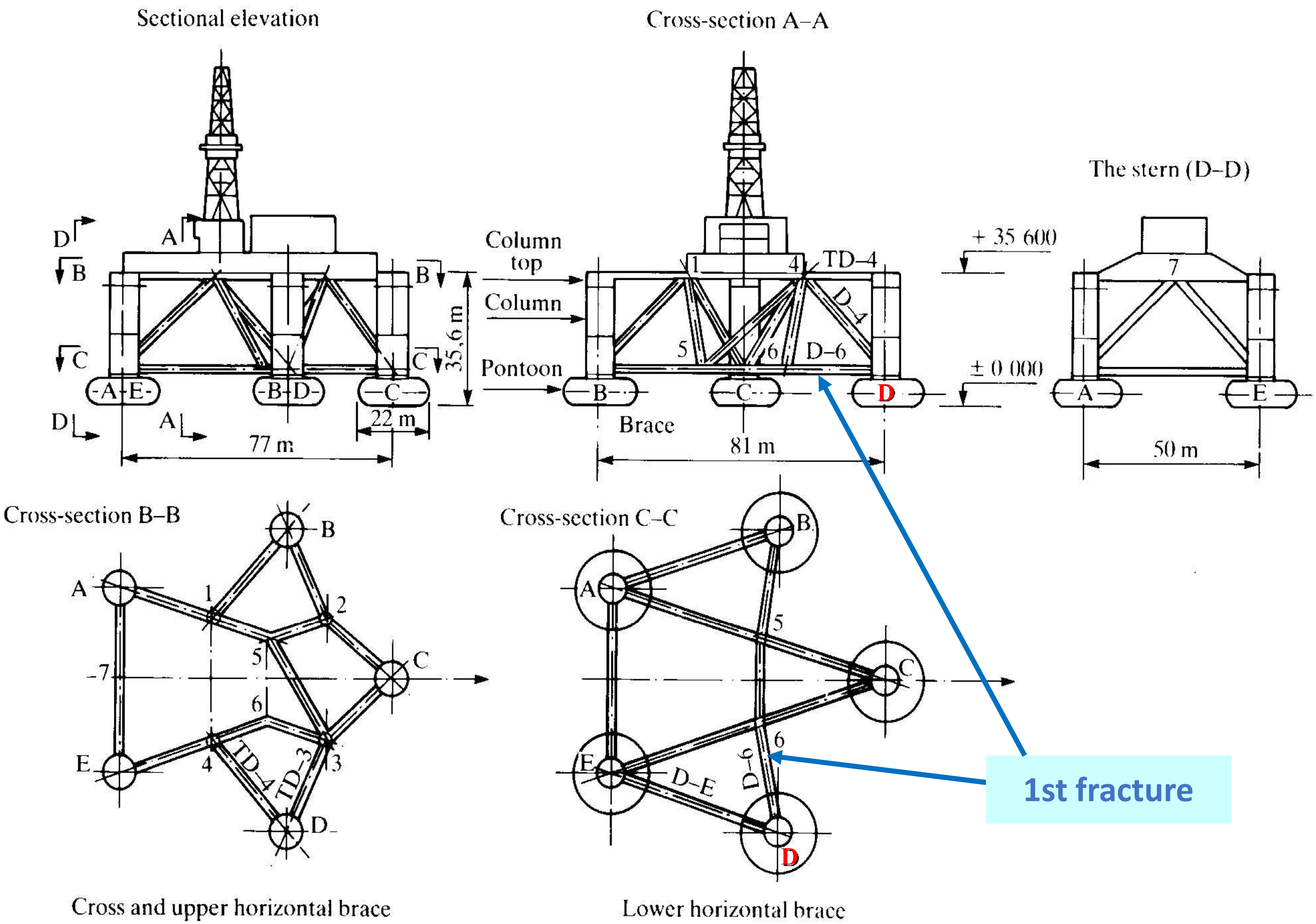
The presence of ***beach marks*** and striations

proves that fatigue caused the fracture.

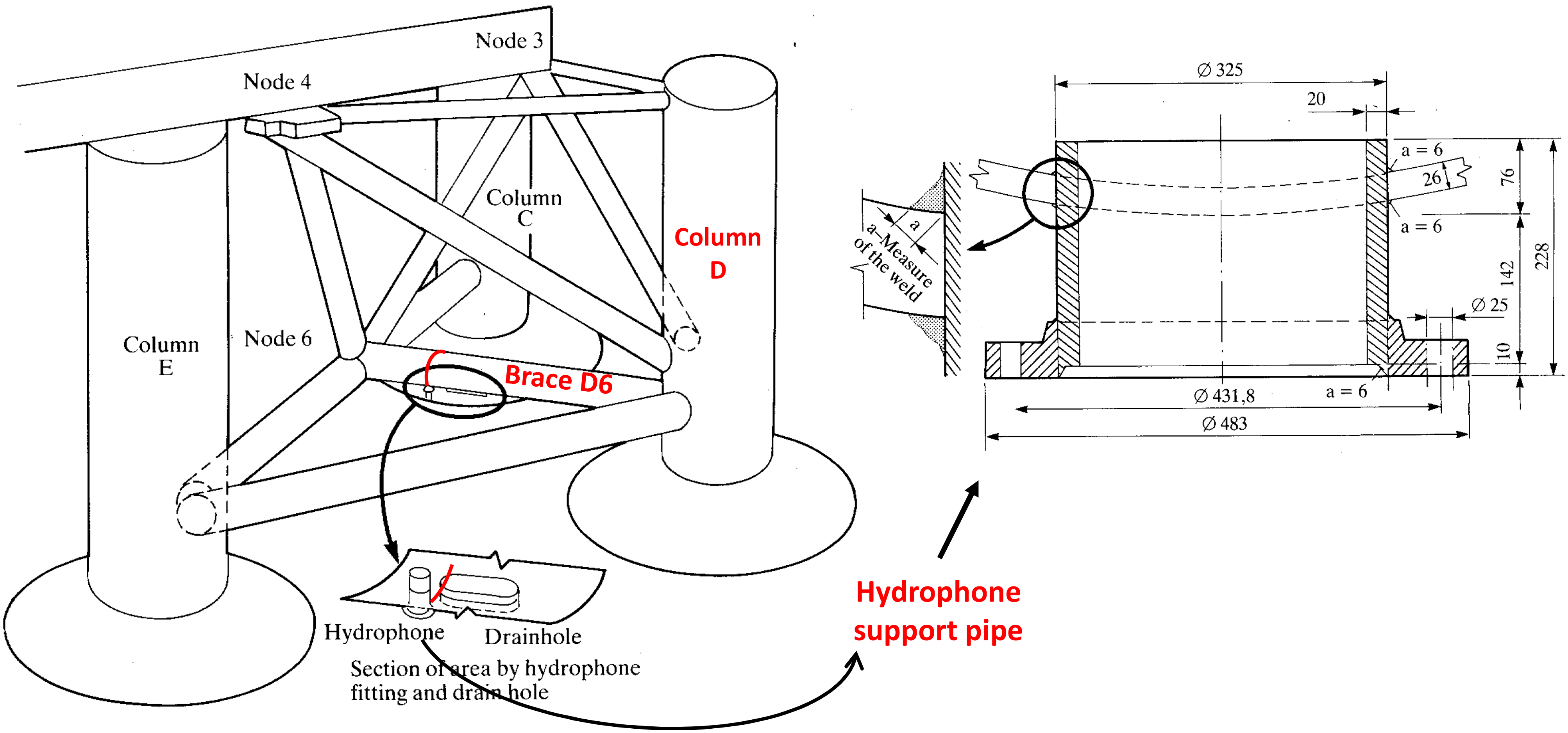


ALK structural arrangement

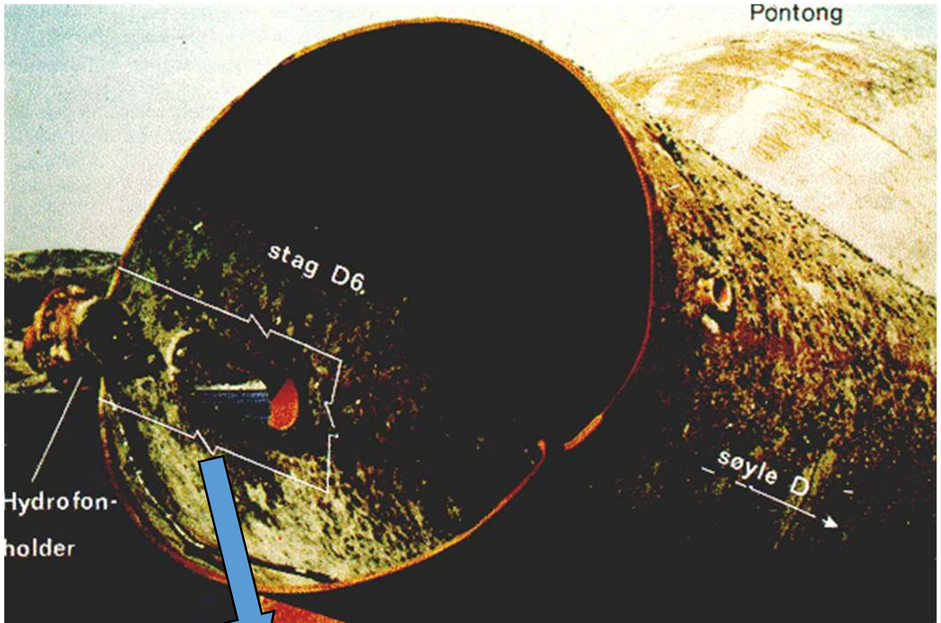
Pentagone design

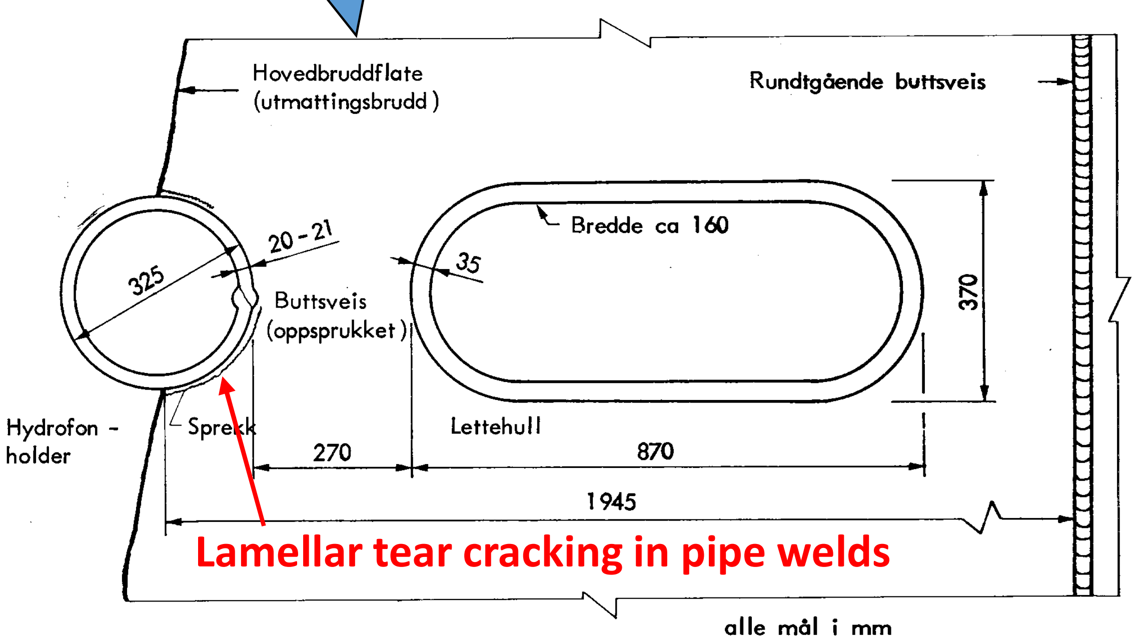


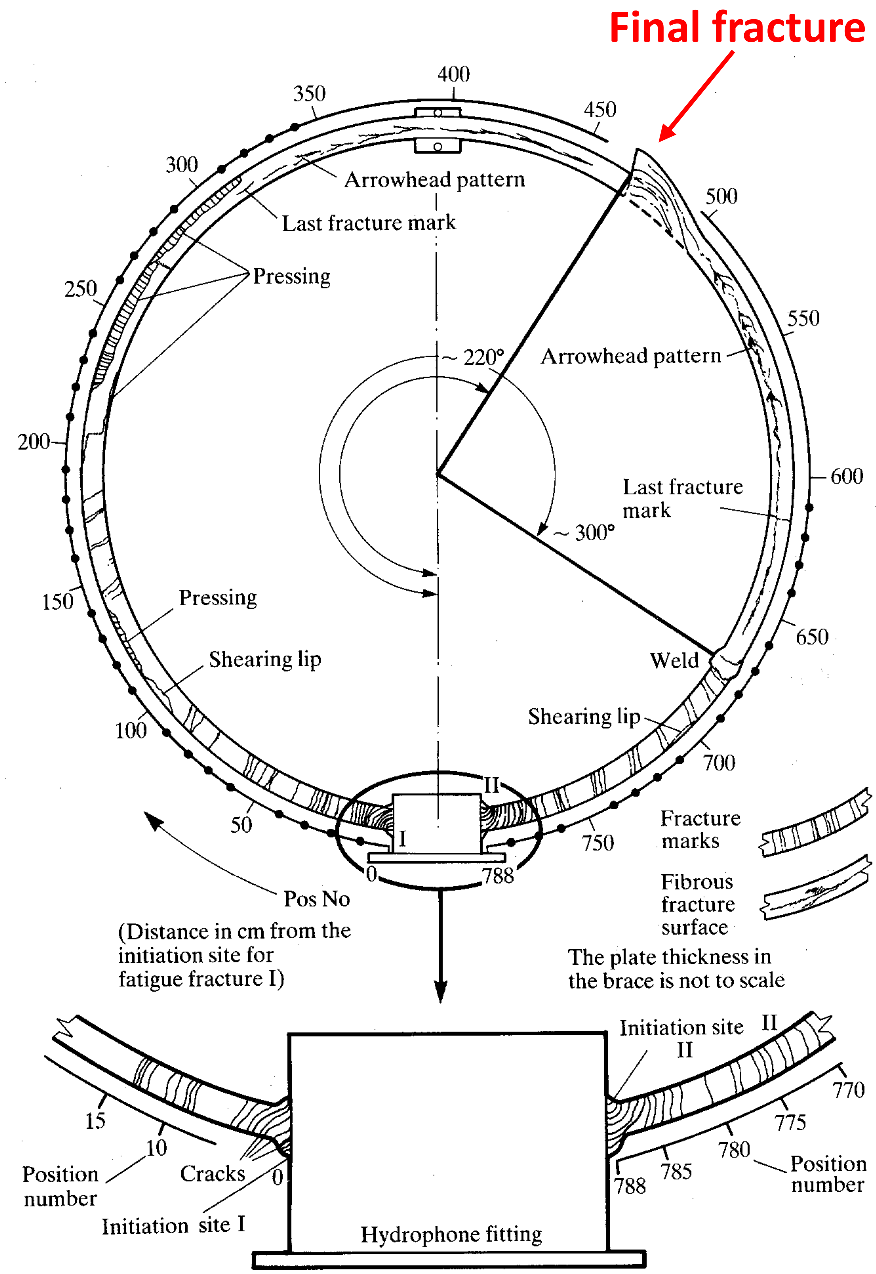
Brace D6 and hydrophone support tube



Fracture in Brace D6



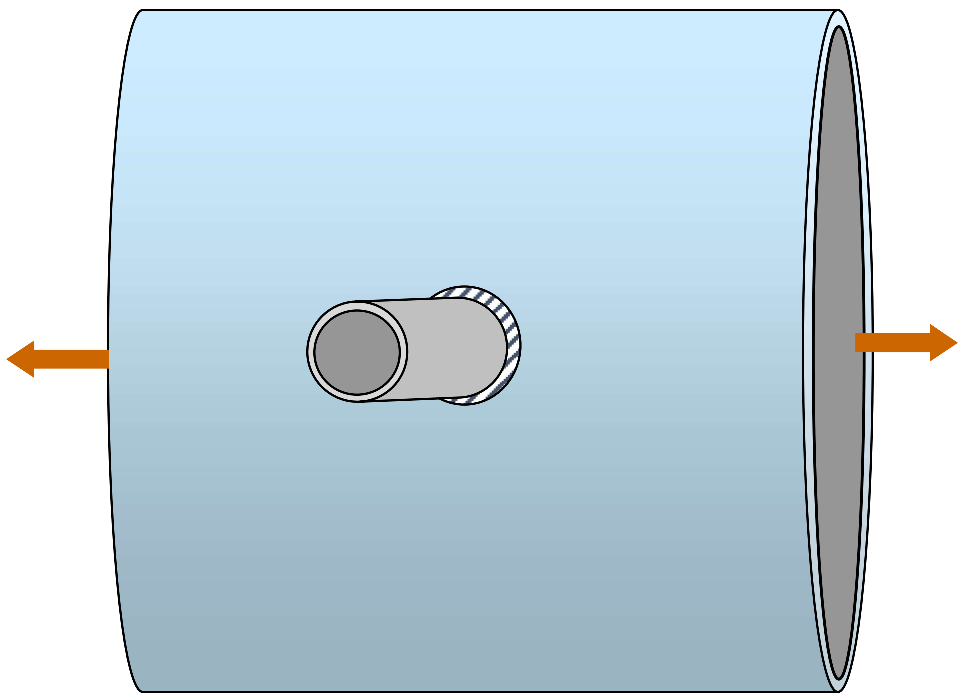




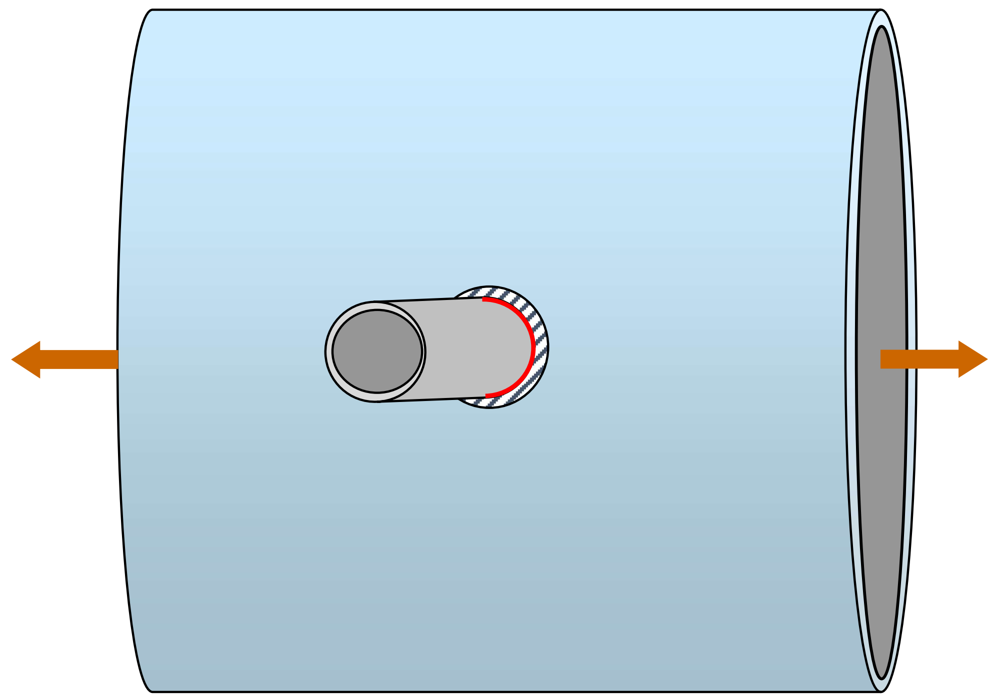
Crack initiation in D6 at support pipe

When the weld around the support pipe is **uncracked**, the stress concentration factor at the weld is 1.6

When the weld around the support pipe is **cracked**, the stress concentration factor at the weld is 3.0, i.e. stress is almost doubled

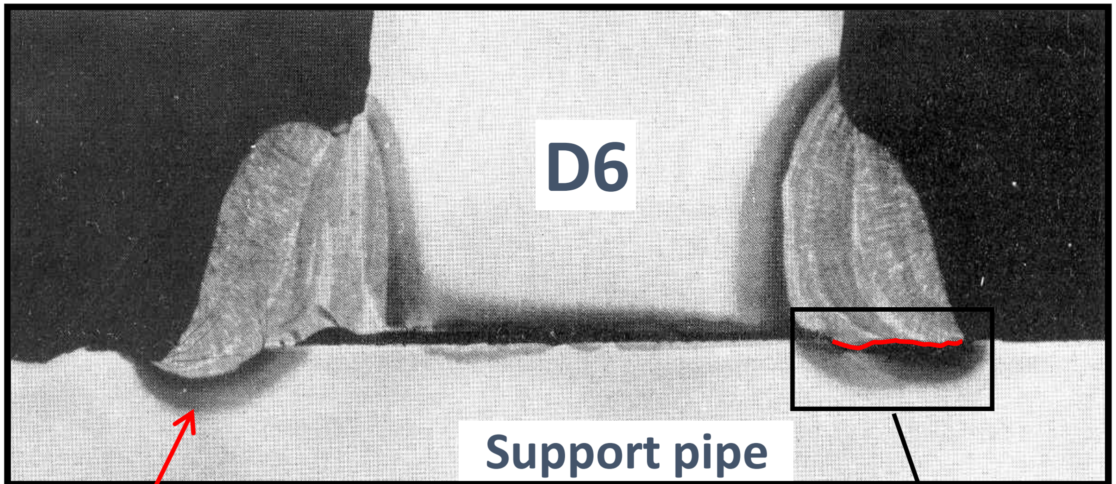


Weld intact: SCF= 1.6

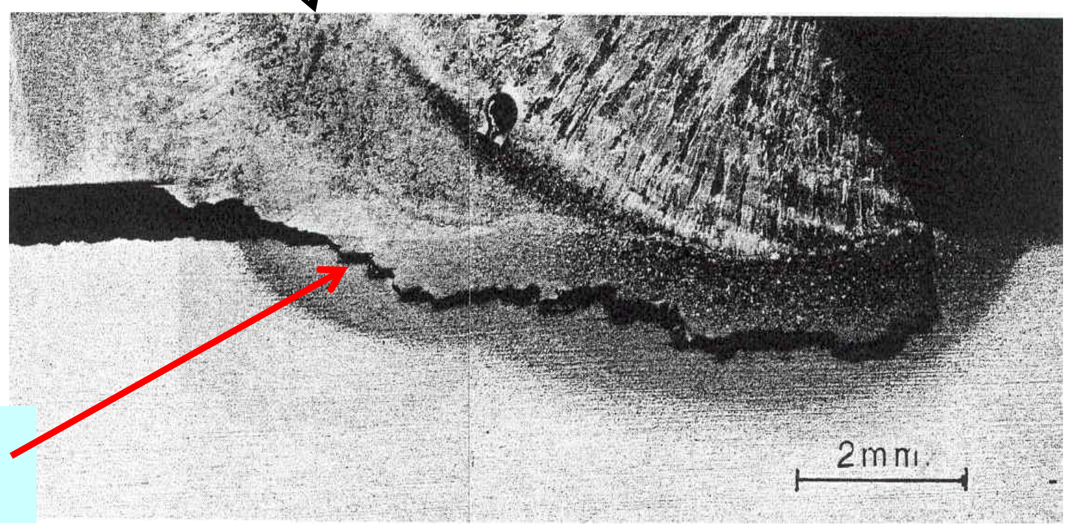


Weld fractured: SCF = 3.0

Lamellar tear cracking

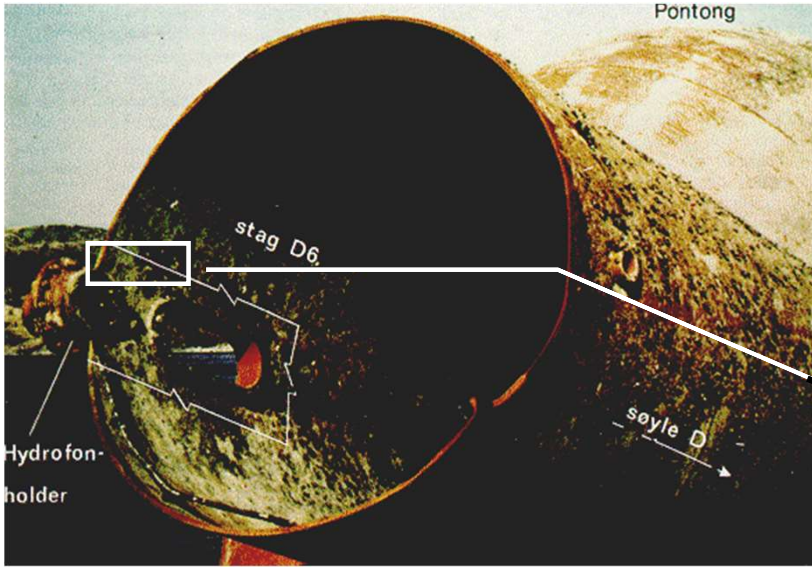


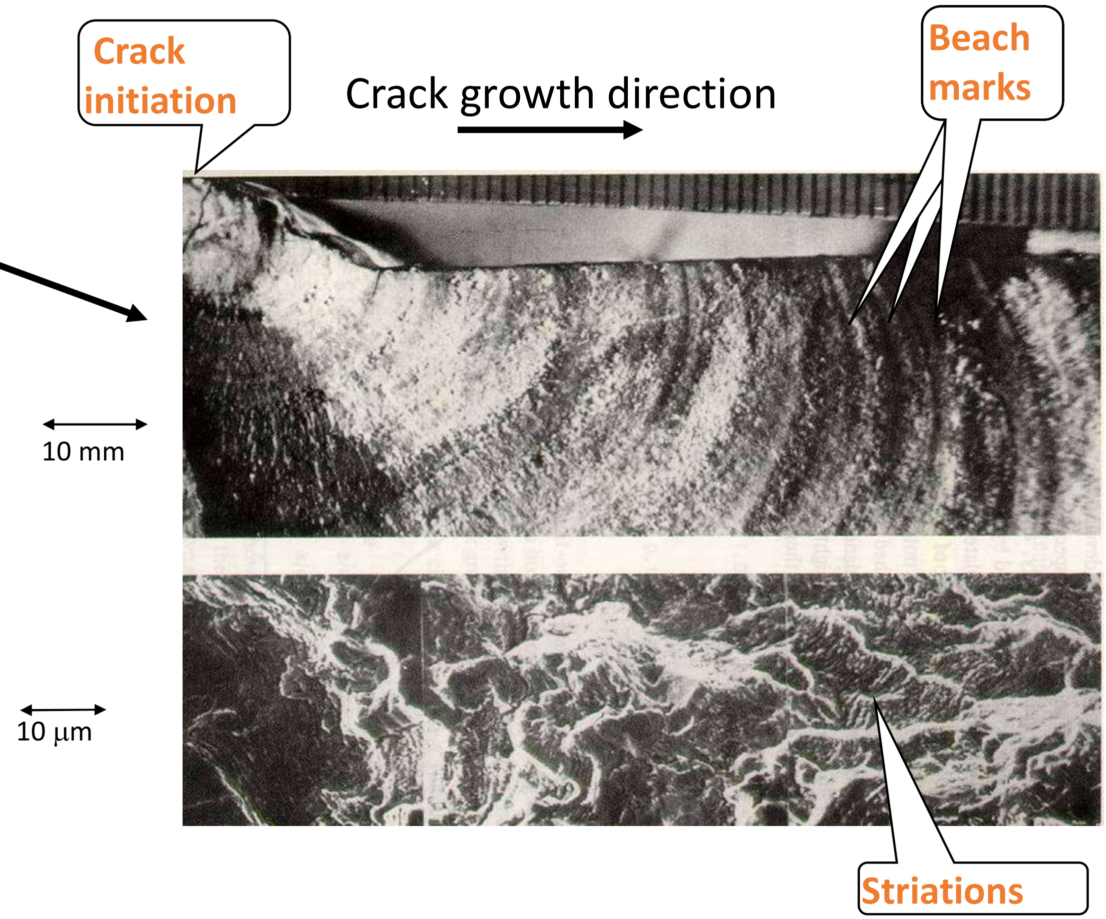
Small penetration



Lamellar tear crack

Crack initiation in D6 at support pipe





Beach marks are lines visible to the naked eye, indicating changes in loading or corrosion conditions.

Striations indicate start-stop positions of the crack tip.

Beach marks and striations prove that fatigue caused the fracture.

Figure 1 Overview jacket

Figure 4 Crack, location A (see Figure 3)

Figure 5 Crack, location A (see Figure 3)

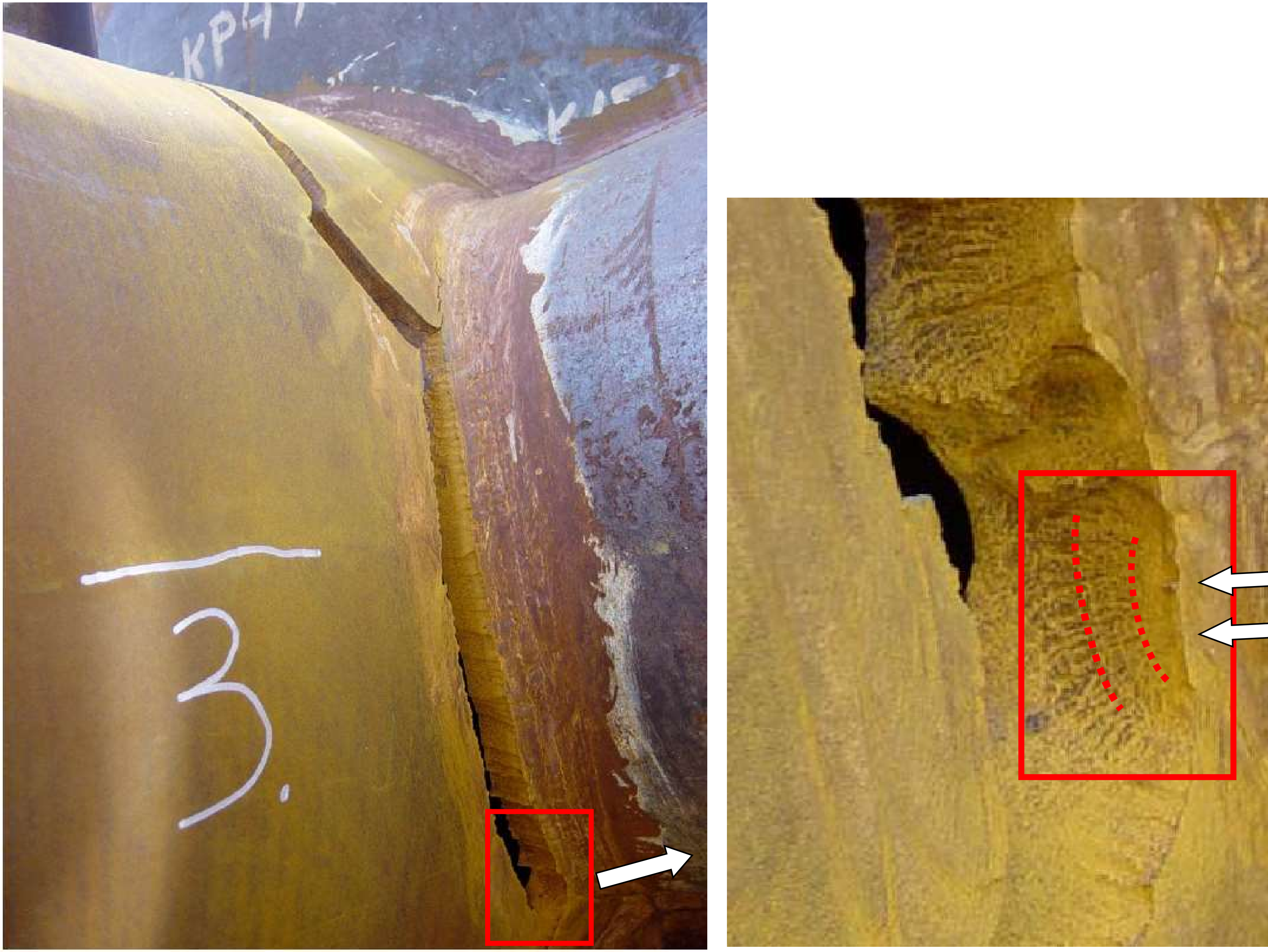


Figure 6 Crack, location A (see Figure 3)

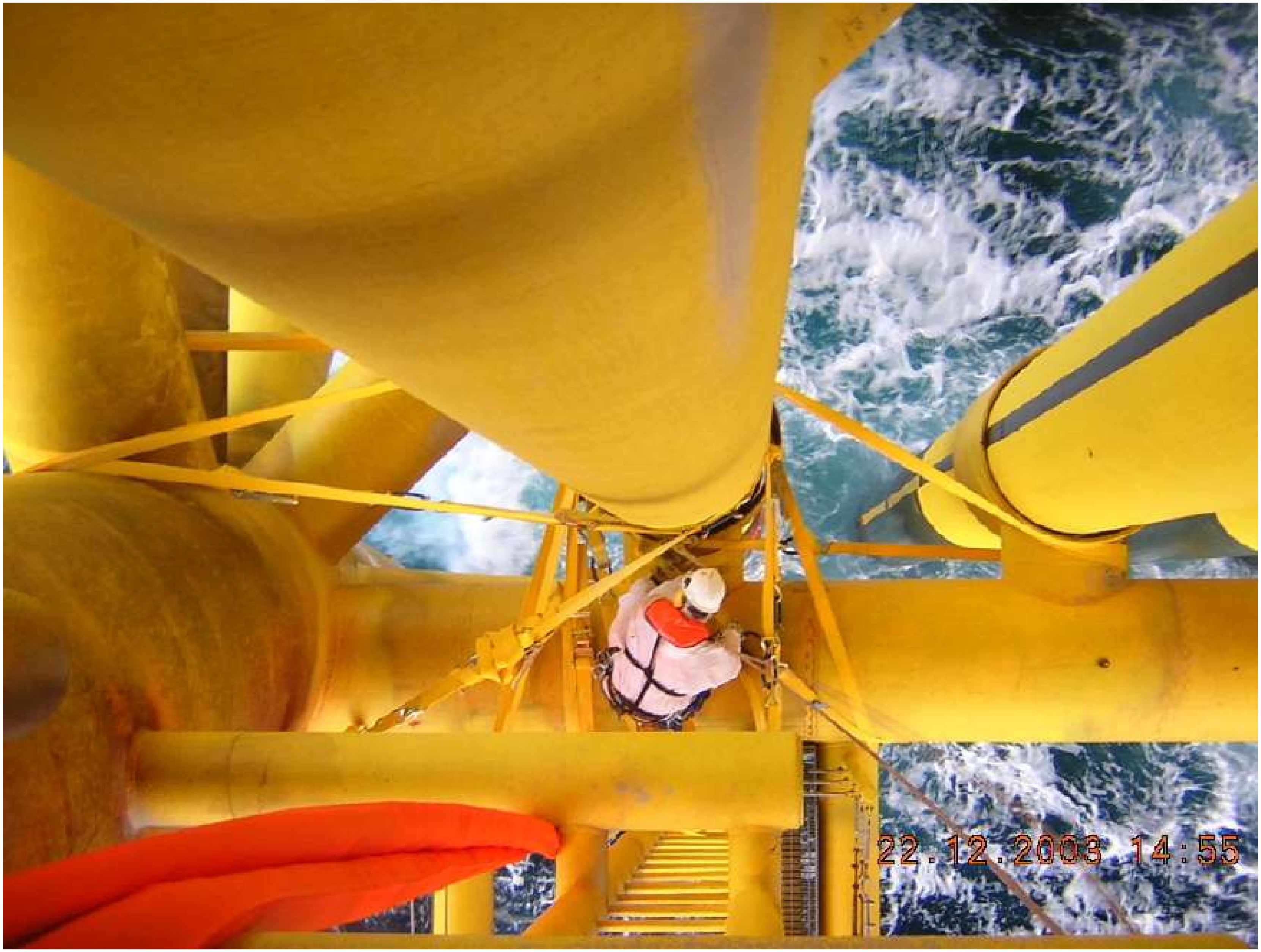
The fractures

Worries:

- **Extent of damage**
- **Causes of cracking**
- **Repairs**
- **Similar structures**

Kvitebjørn fatigue problems

Fatigue cracks in guide pipes for production risers

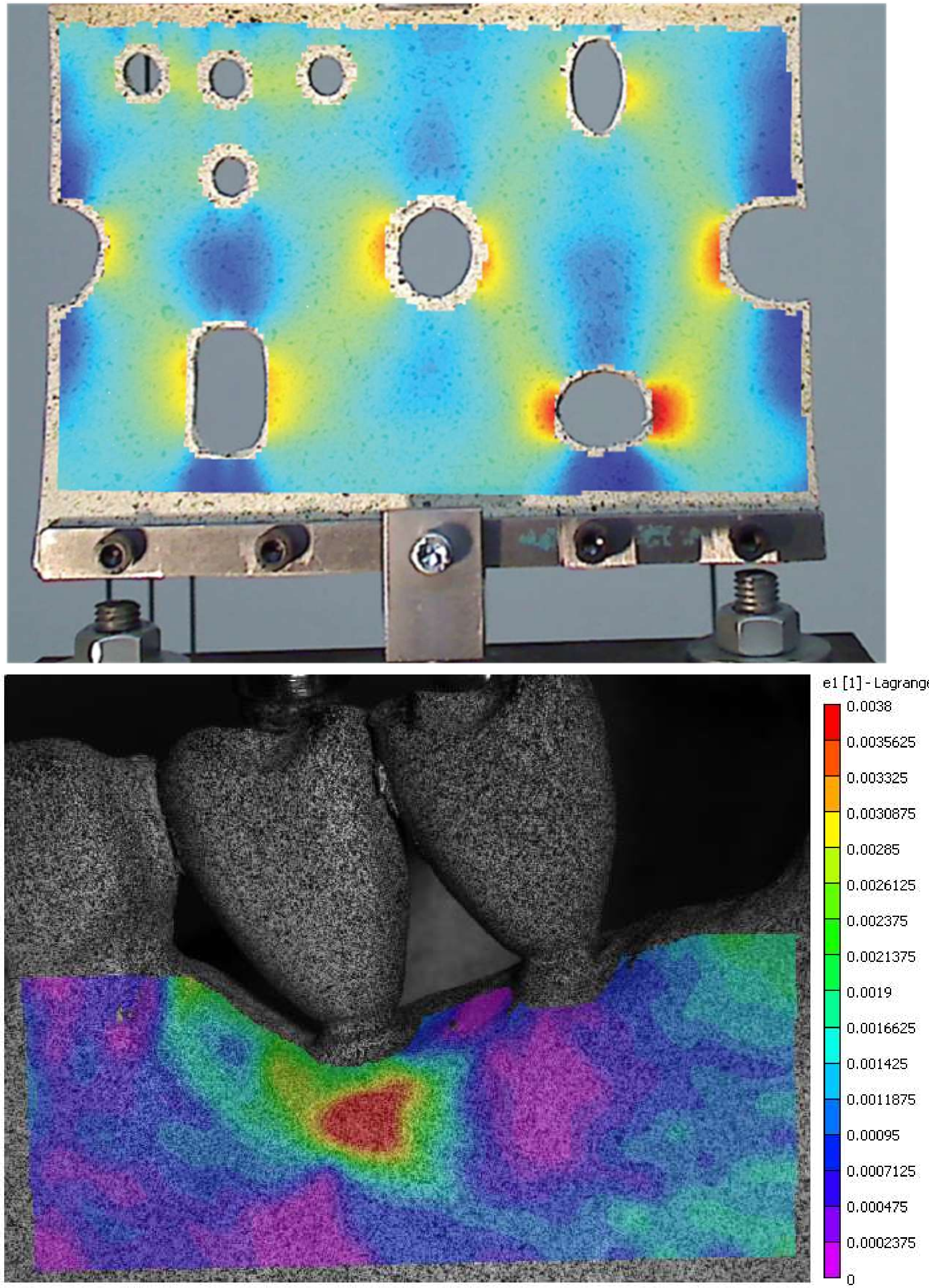

22.12.2003 14:55


U.S. ARMY


FEELS LIKE
PIONEER SUSPENSION


FEELS LIKE
PIONEER SUSPENSION


Crack
Crack


e1 [1] - Lagrange
0.0038
0.0035625
0.003325
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0.00285
0.0026125
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0.0021375
0.0019
0.0016625
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0.0007125
0.000475
0.0002375
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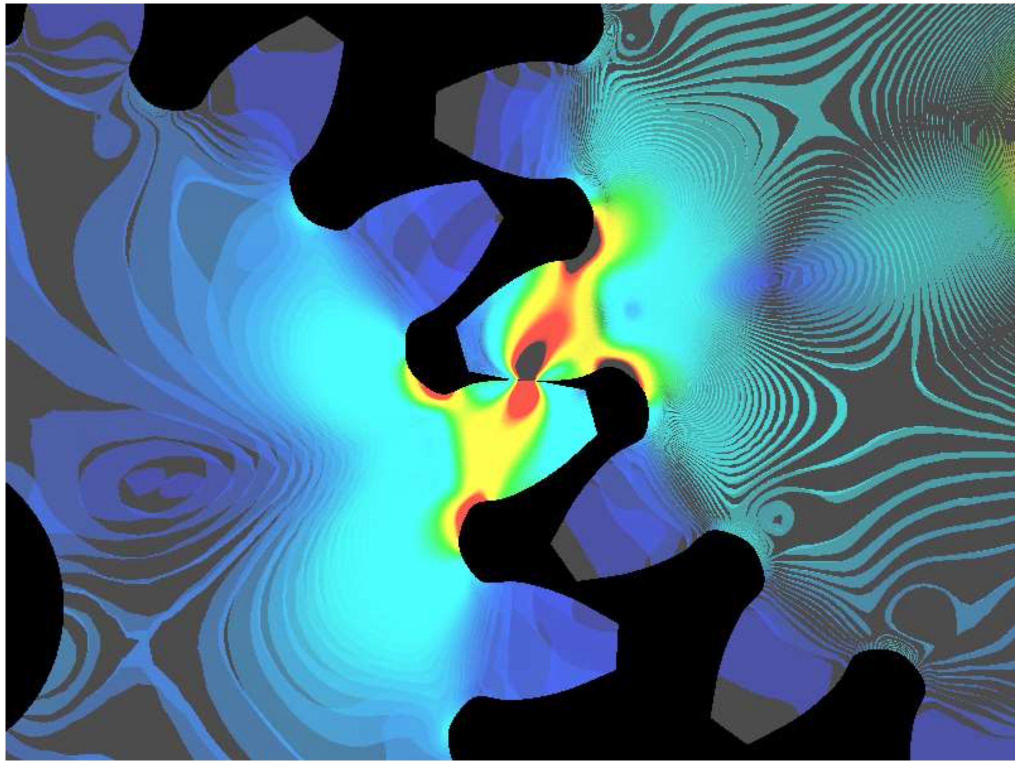
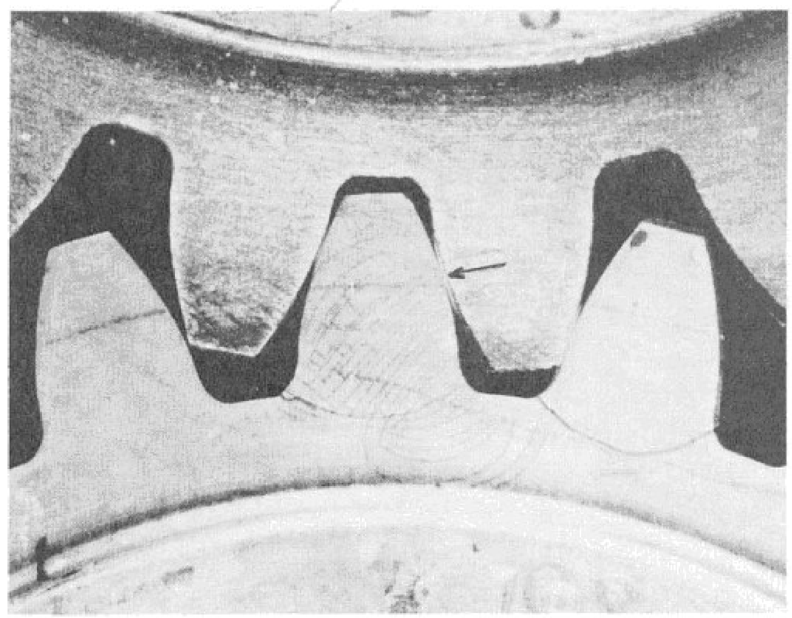
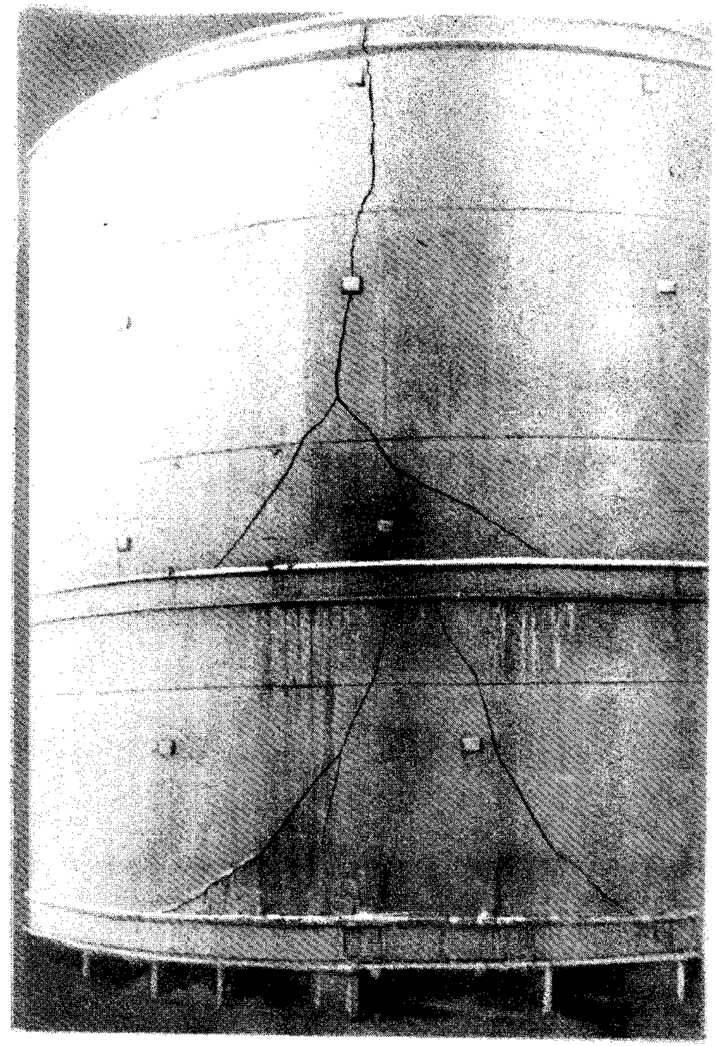
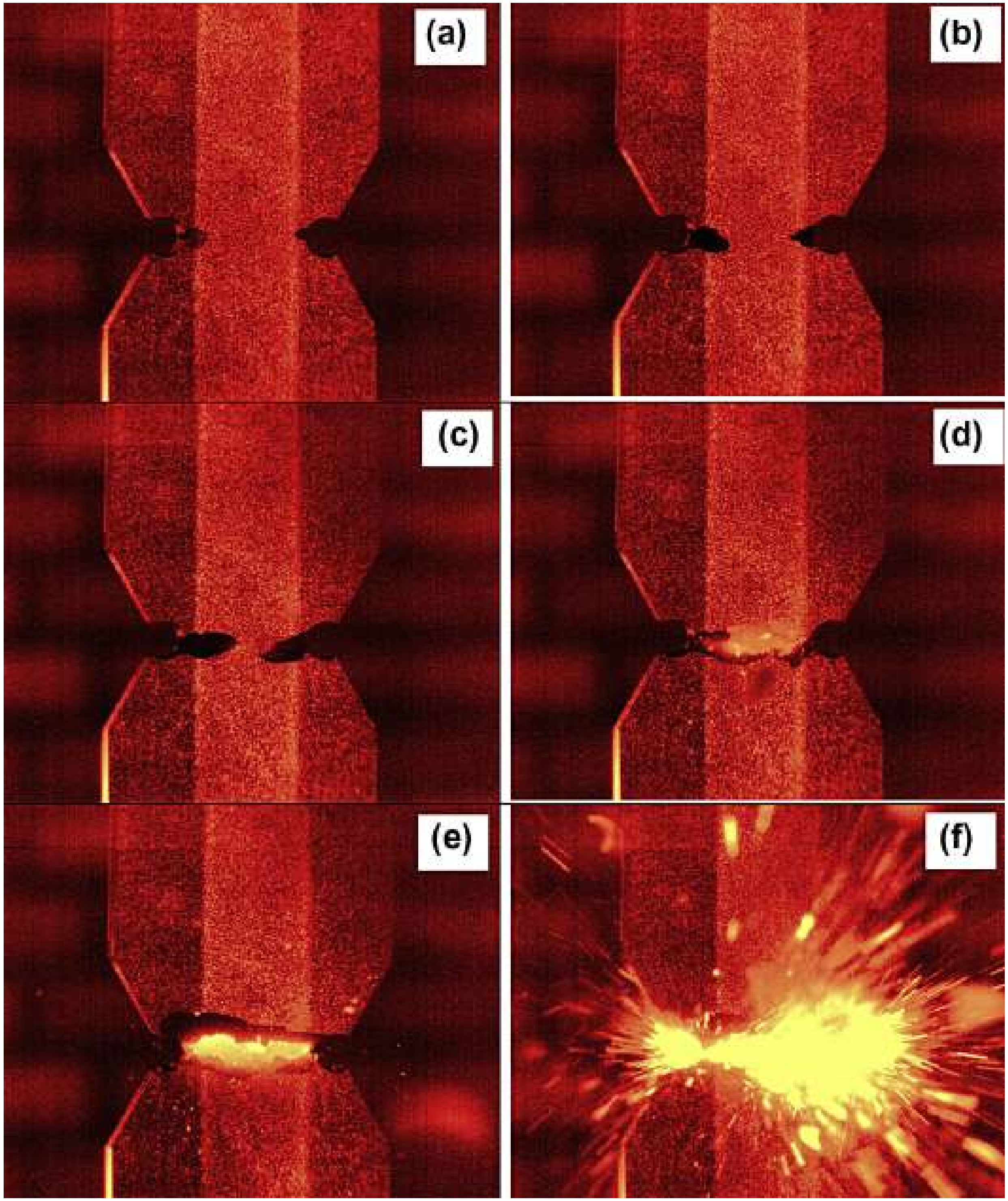






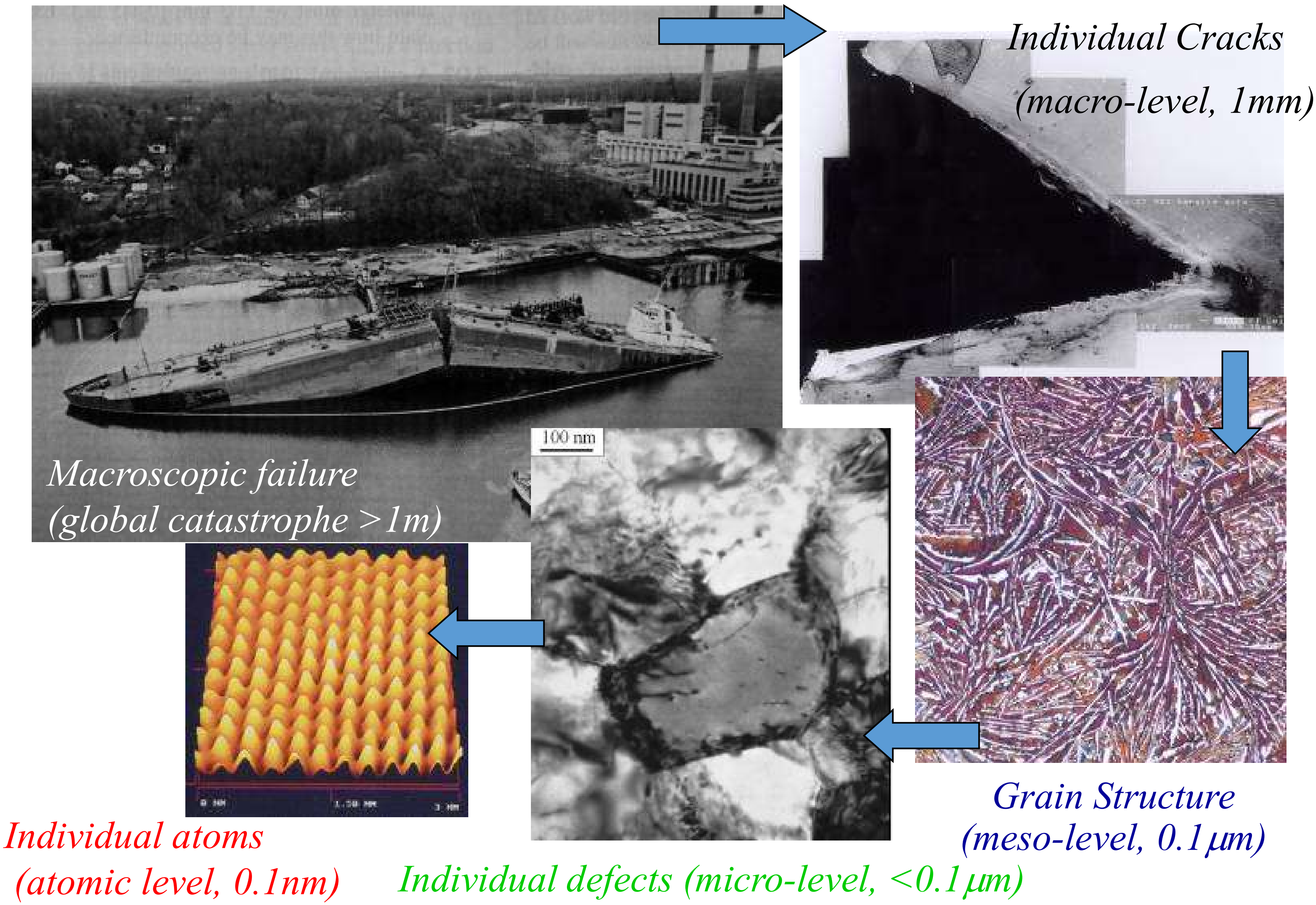
7. Accident to a drying tower at the Cherepovets chemical plant (December 1977).

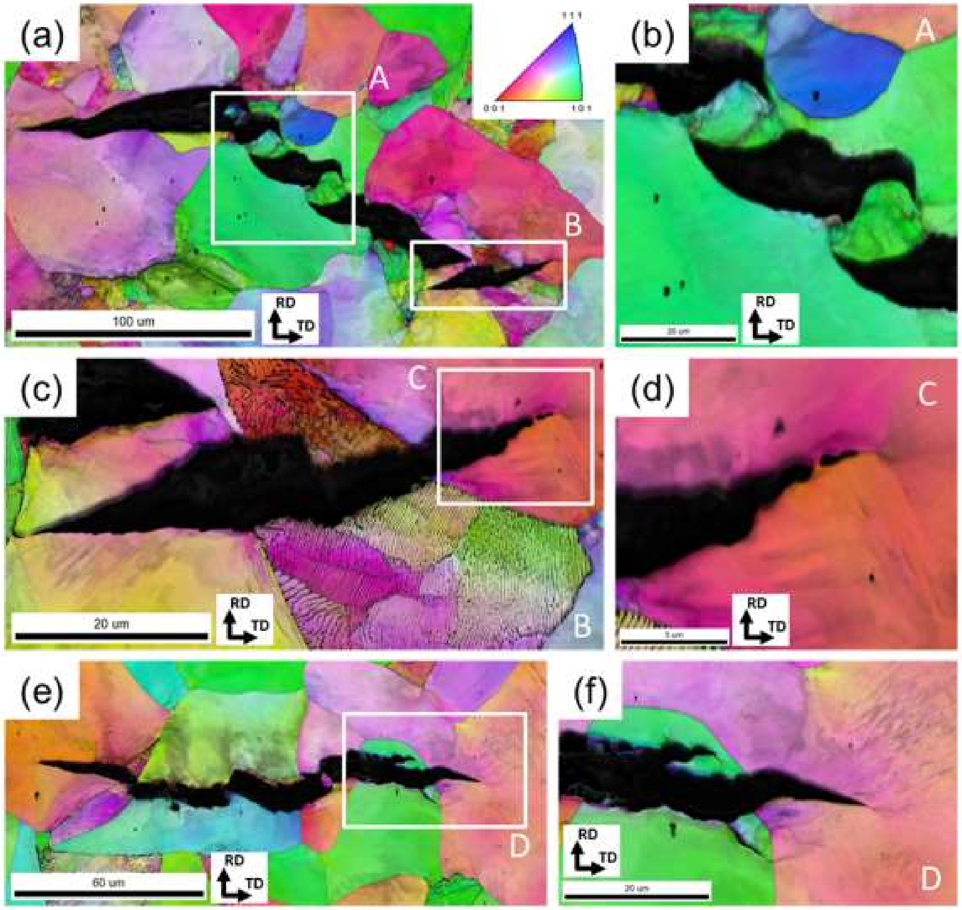


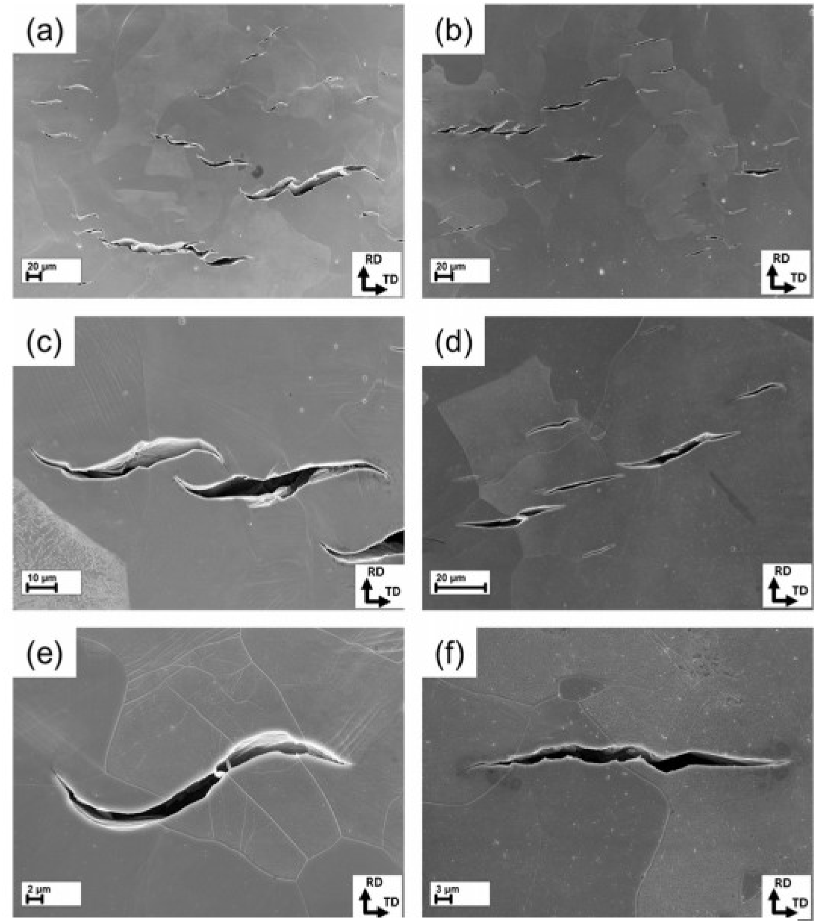

(a)
(b)
(c)
(d)
(e)
(f)

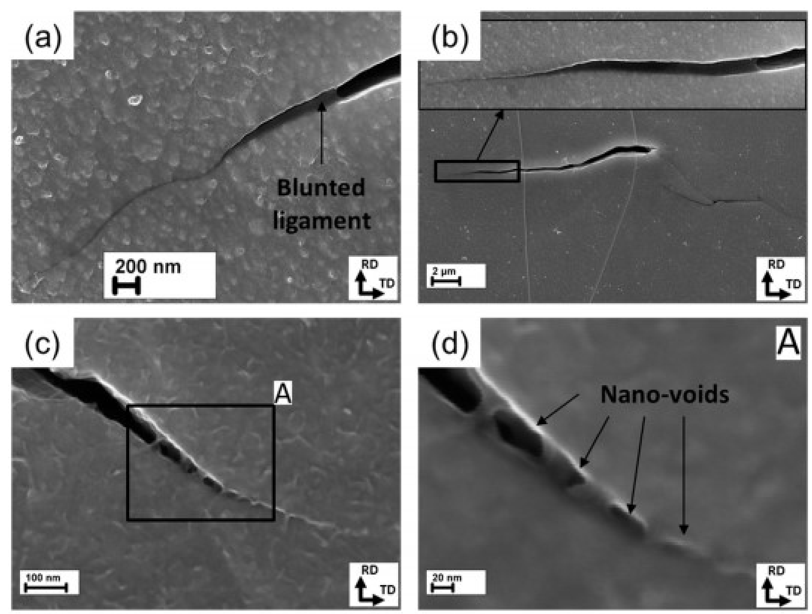
FATIGUE IS LOCAL

Multi-Scale Nature of Fracture







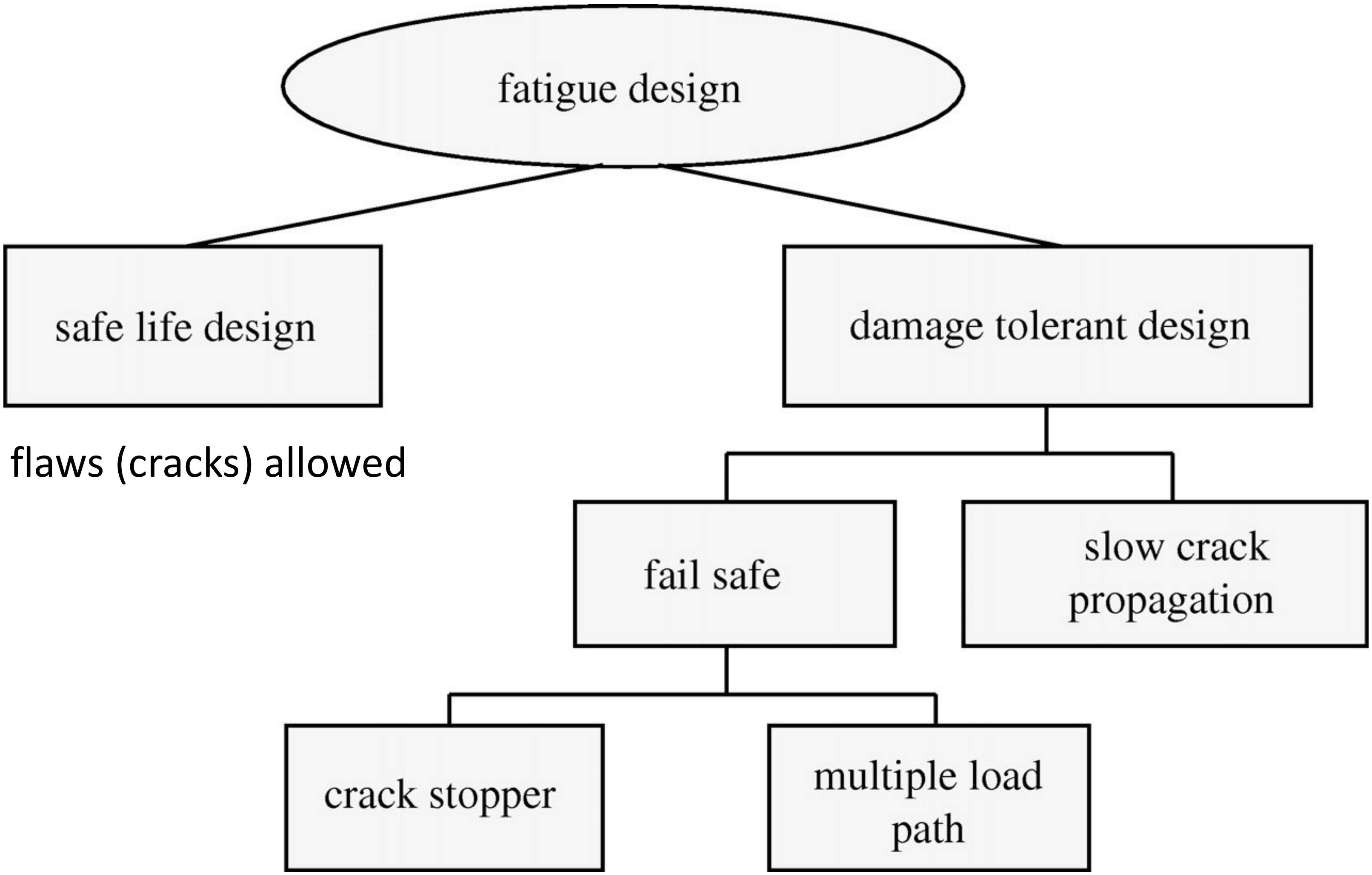


Merson et al. Engineering Fracture Mechanics 214 (2019) 177–193

If you have a ‘cold’ you can go to a cold you can go to your family doctor if you have a ‘fatigue problem’ you need a specialist

Fatigue is a very local phenomenon and a structure of several meters in length can fail for a crack or a defect less than 1 mm!!!

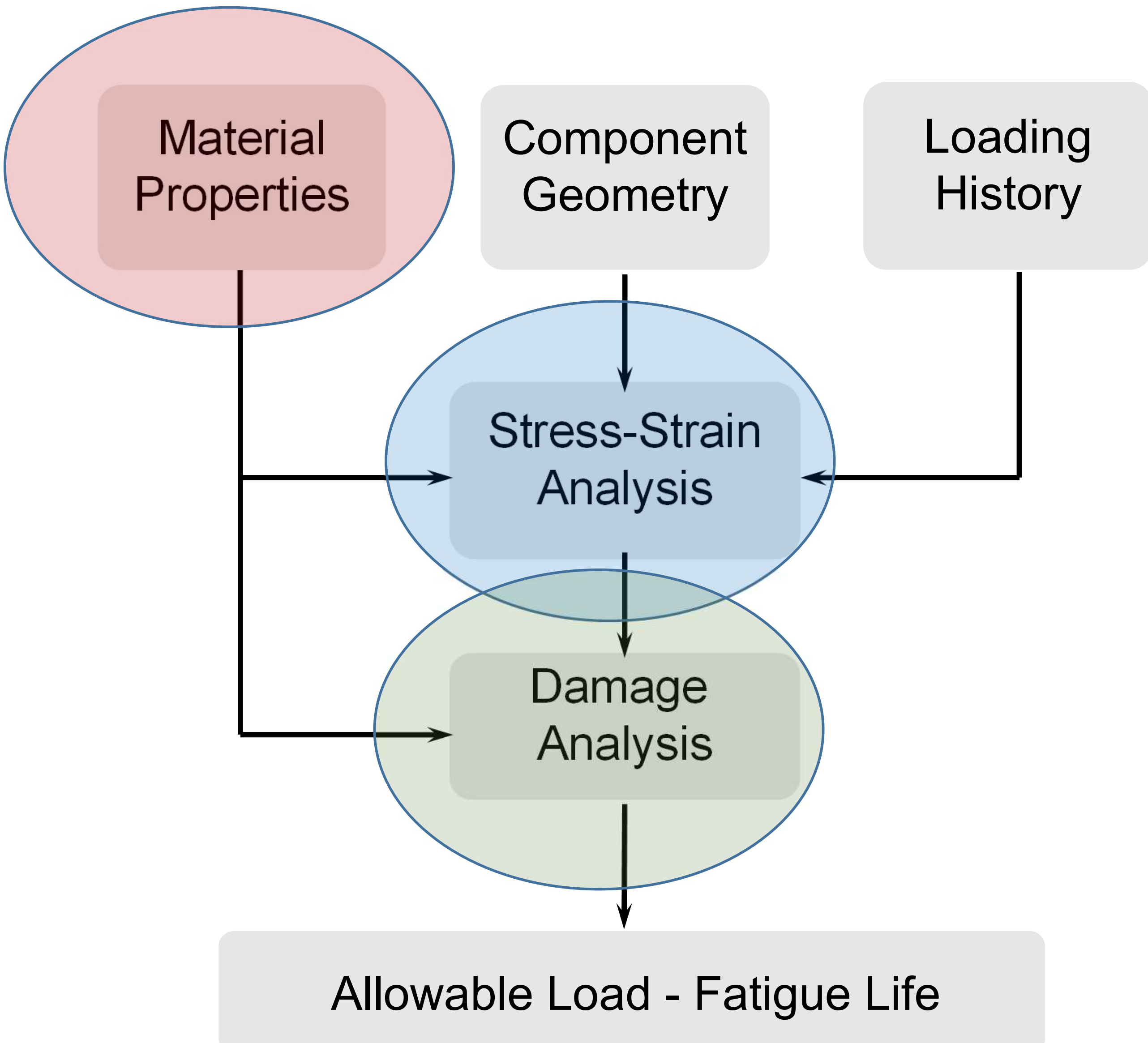
Fatigue design



no flaws (cracks) allowed

R. I. Stephens, A. Fatemi, R. R. Stephens, H. O. Fuchs, Metal Fatigue in Engineering, Wiley

Stress (strain)-based fatigue design



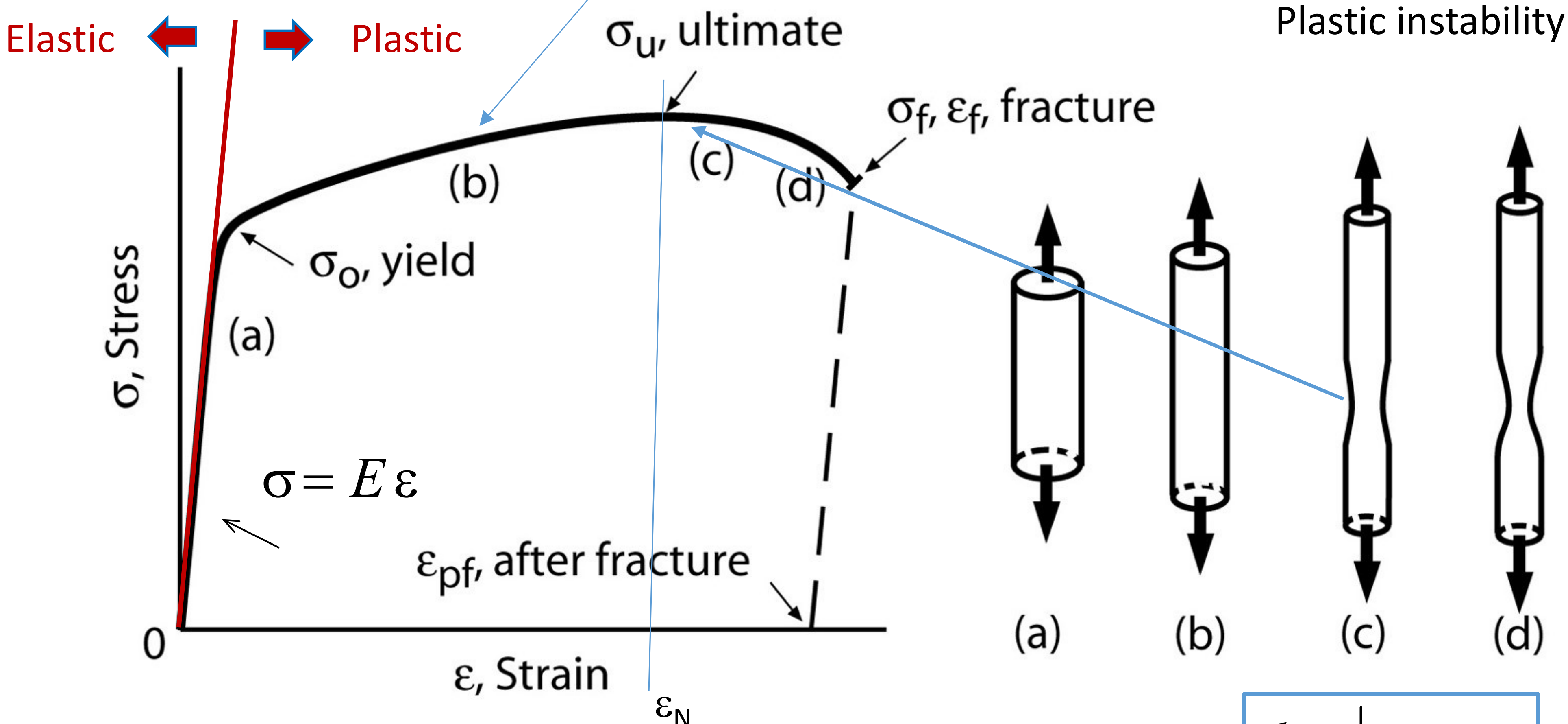
Fracture Mechanics
Pre-requisites

Materials Properties

$$n = \frac{d\log(\sigma)}{d\log(\varepsilon)} = \frac{\varepsilon}{\sigma}\frac{d\sigma}{d\varepsilon}$$ empiric

- Tensile

$$\sigma(\varepsilon) = K\varepsilon_p^n$$



$$\varepsilon_T(\varepsilon_N) = n$$

$$\left.\frac{d\sigma_T}{d\varepsilon_T}\right|_{\varepsilon_N} = \sigma_T$$

Learn from Tensile

- Elastic and Plastic stress-strain components
- Key Properties - E, YS, UTS, ε_f, ε_N
- Hardening law
- Plastic Instability

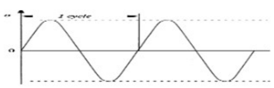
Fatigue Life

- Two stages
- Two concepts in design

Crack propagation

Stress-based HCF

Crack initiation



Unflawed body

Fatigue life is governed by crack initiation

Strain-based LCF

Crack initiation

Crack propagation

Rupture

Unflawed body

Fatigue life is governed by crack propagation (defect tolerant approach)

Cyclic Deformation

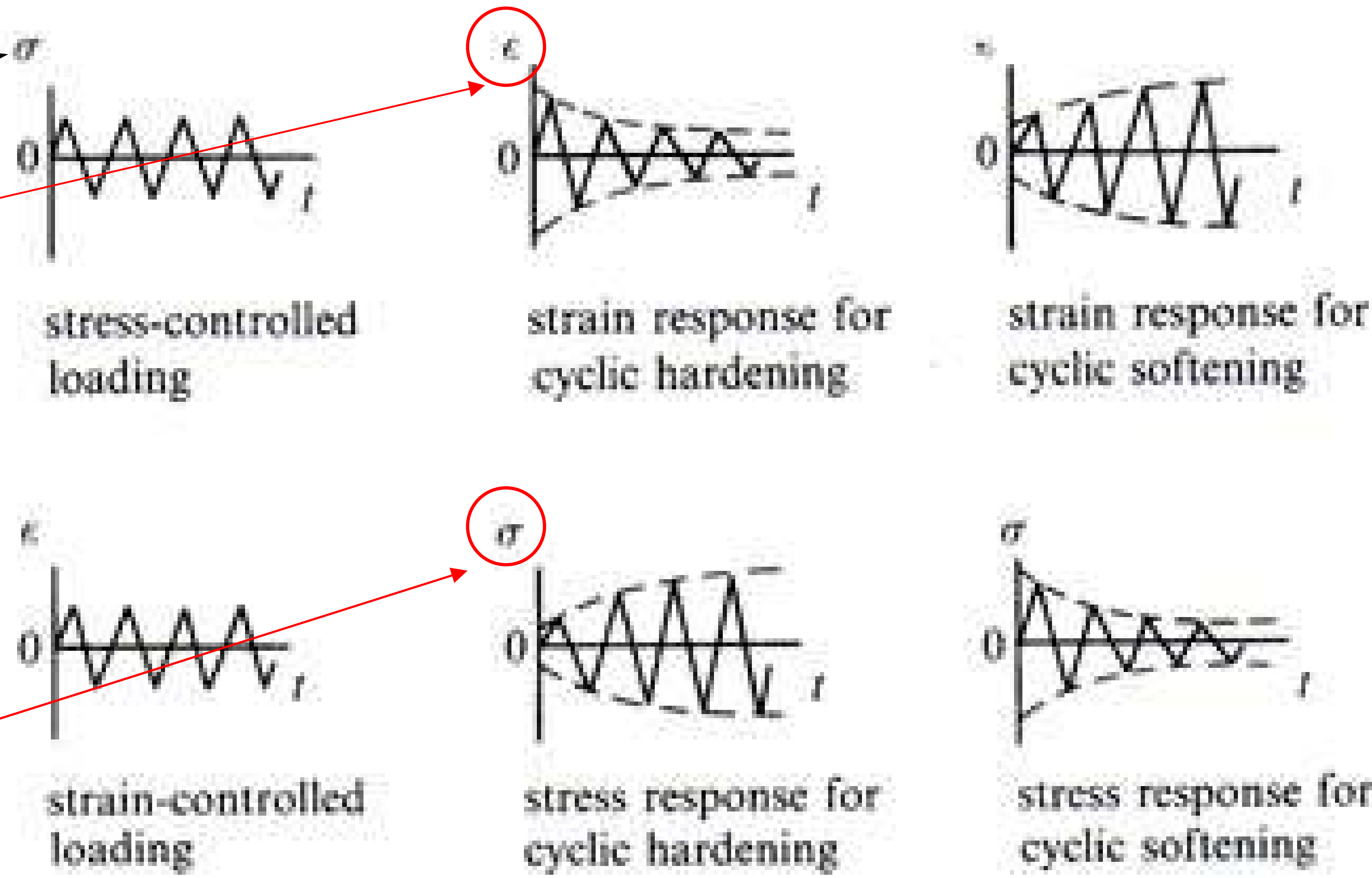
Response

Stress= const

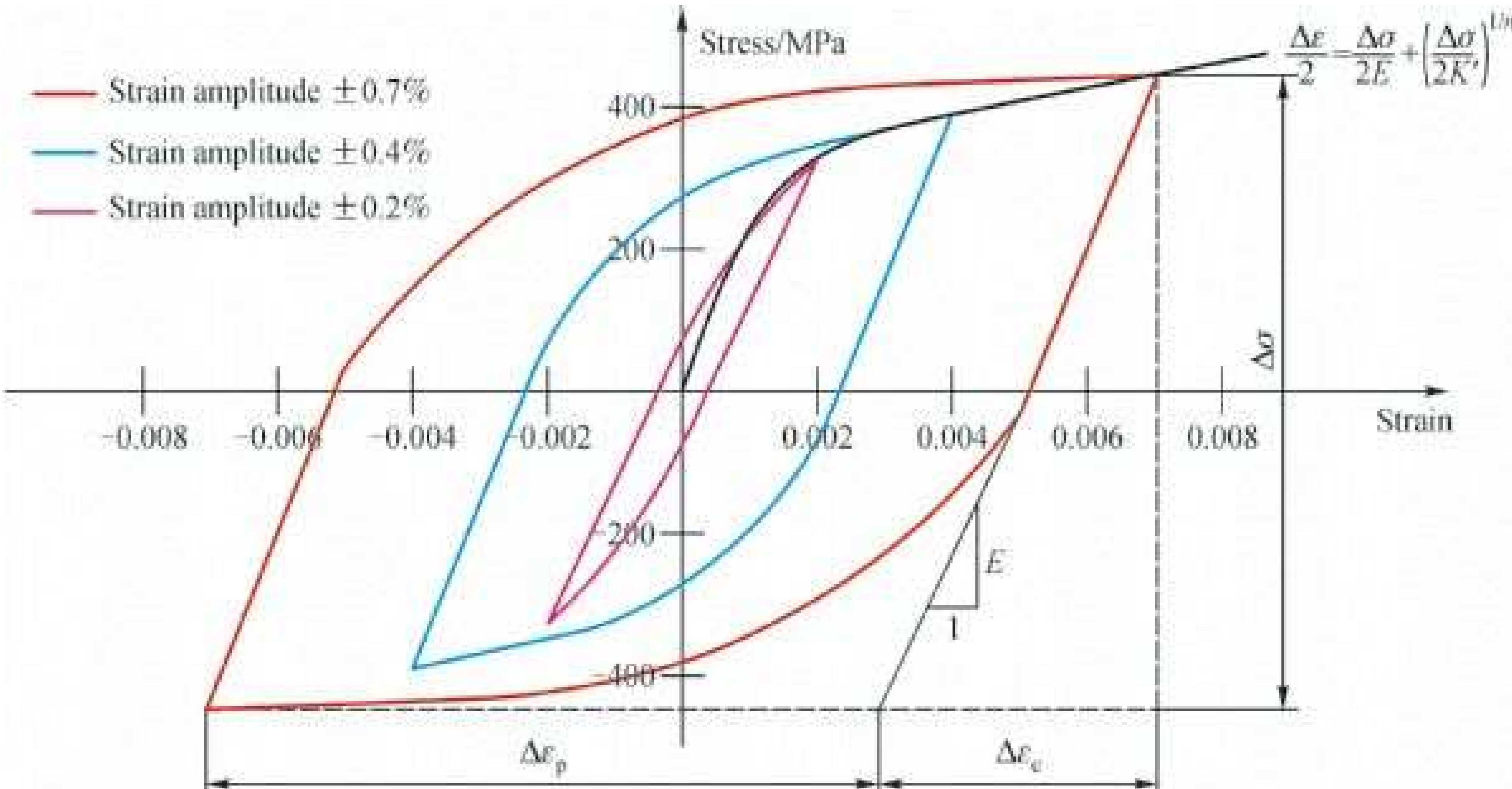
measure **Strain** as a response

Strain (total or plastic)= const

measure **Stress** as a response



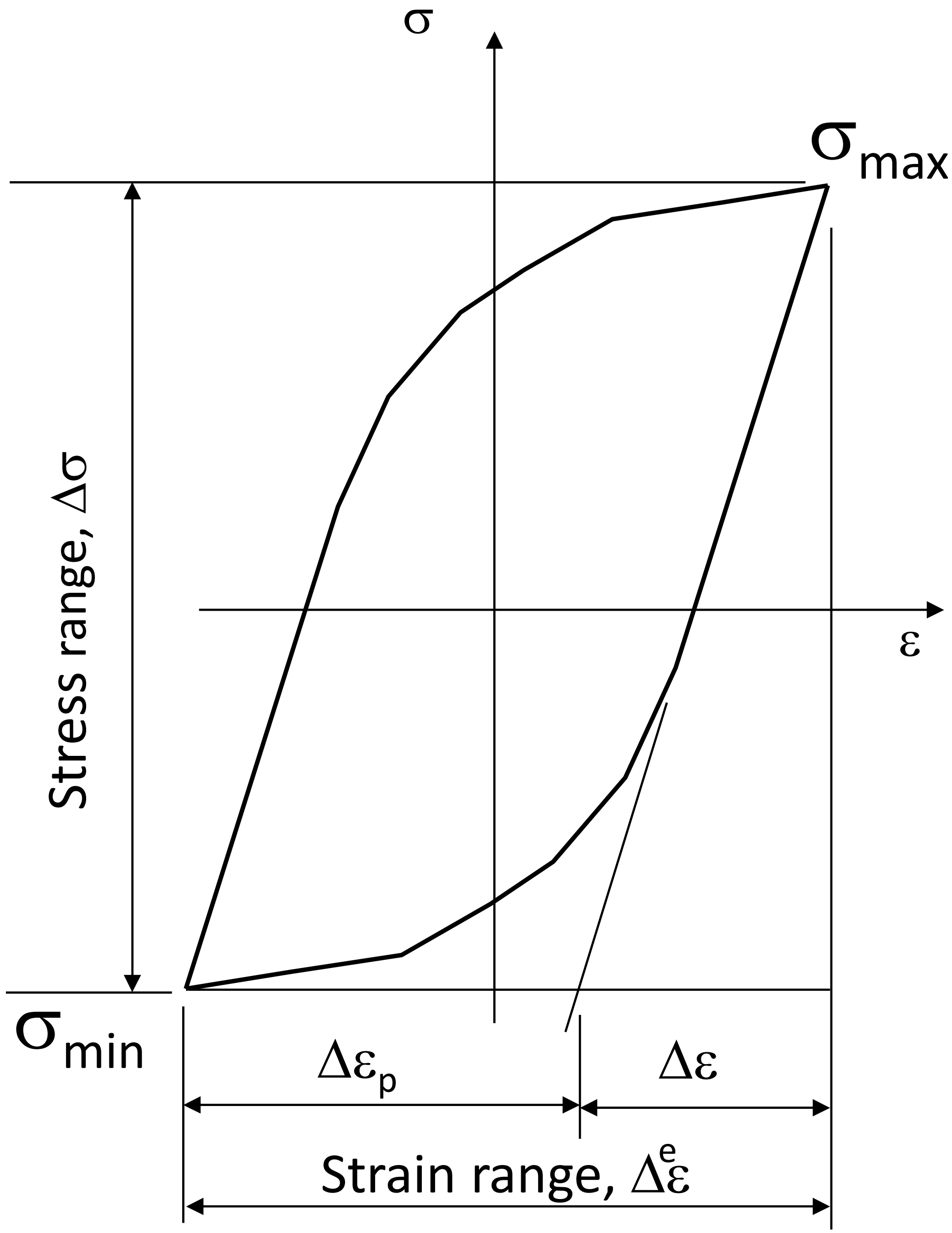
Measurable – Cyclic Hysteresis Loop



cyclic stress-strain curve.

Plastic work → Microstructure + Heat

Cyclic Hysteresis Behaviour



Range

$$\Delta\varepsilon = \Delta\varepsilon_e + \Delta\varepsilon_p$$

$$\Delta\varepsilon_e = \Delta\sigma / E$$

Amplitude

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2}$$

$$\frac{\Delta\varepsilon_e}{2} = \frac{\Delta\sigma}{2E}$$

Equations for Cyclic Stresses

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min} \quad ,$$

stress range

$$\sigma_a = \frac{\Delta\sigma}{2}$$

stress amplitude

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

mean stress

$$\sigma_{\max} = \sigma_m + \sigma_a \quad , \qquad \sigma_{\min} = \sigma_m - \sigma_a$$

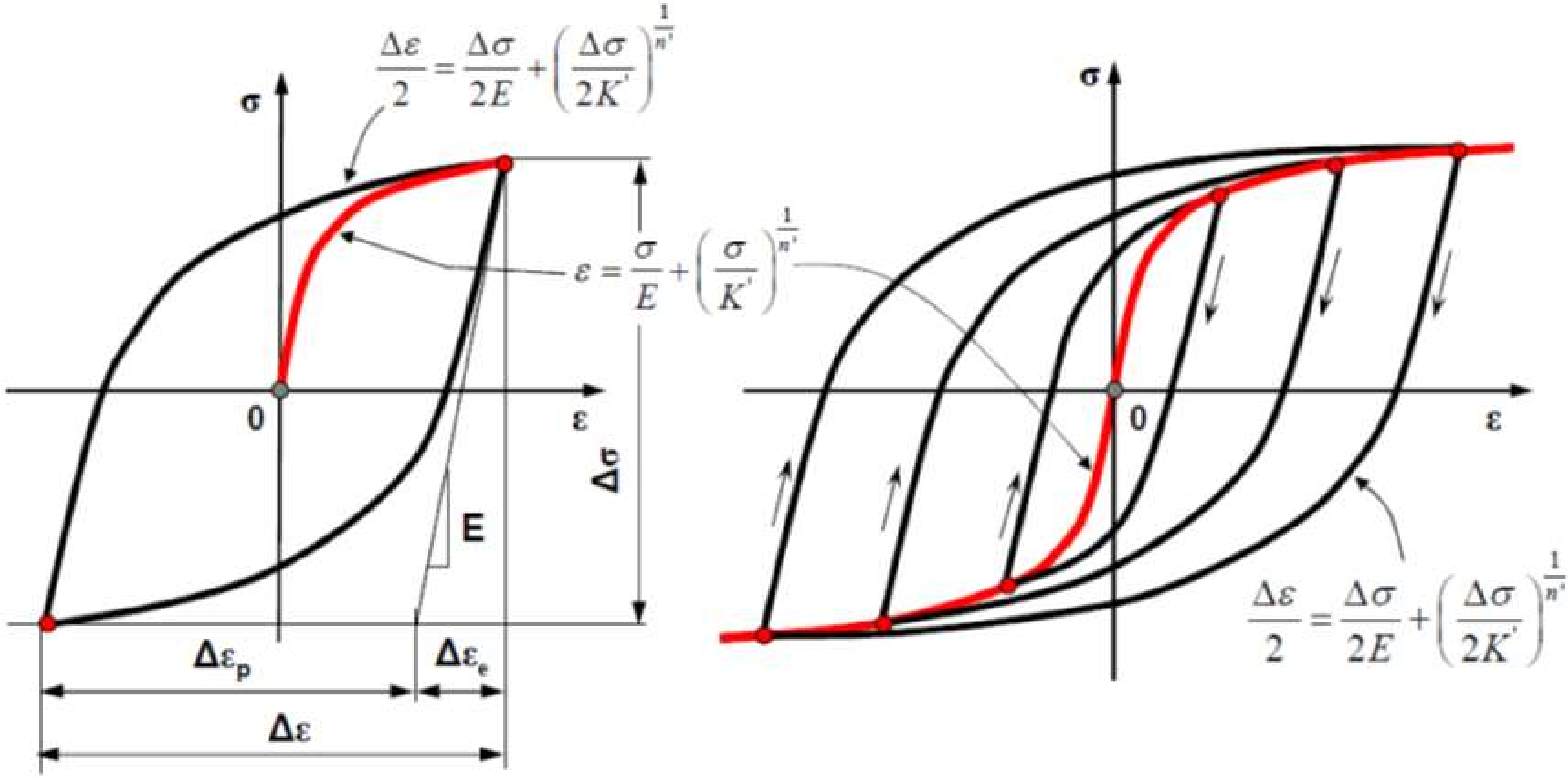
min, max stress in a cycle

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \quad ,$$

Stress ratio

$$\Delta\sigma = \sigma_{\max}(1 - R)$$

Equations for the cyclic hysteresis loops and CSSC



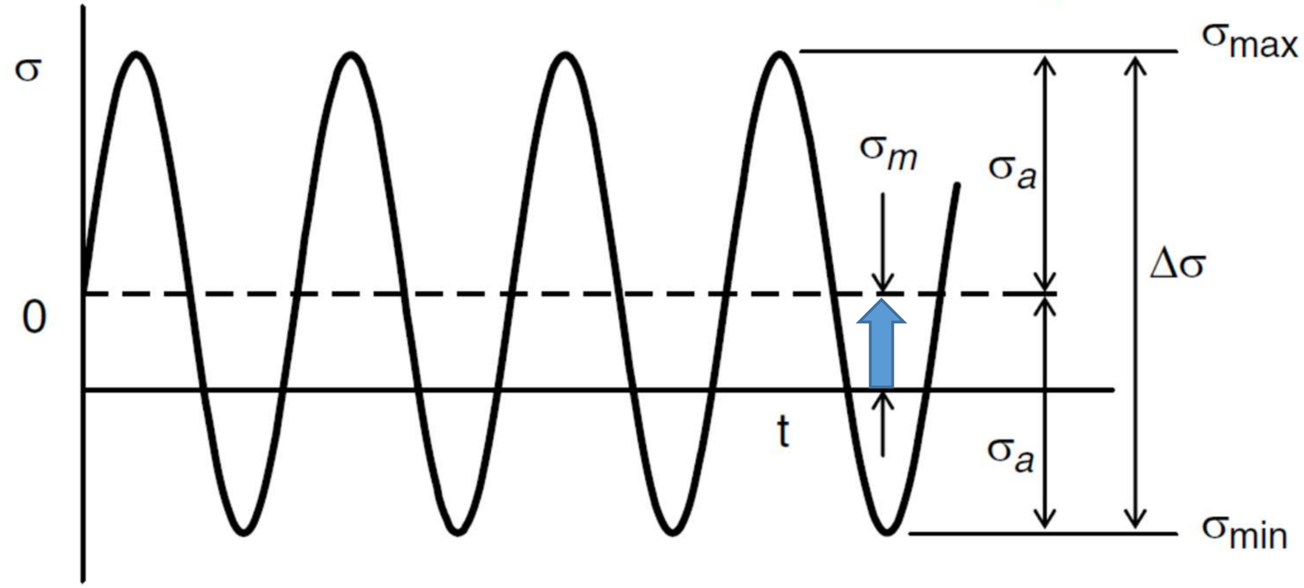
Equation of the cyclic stress-strain curve

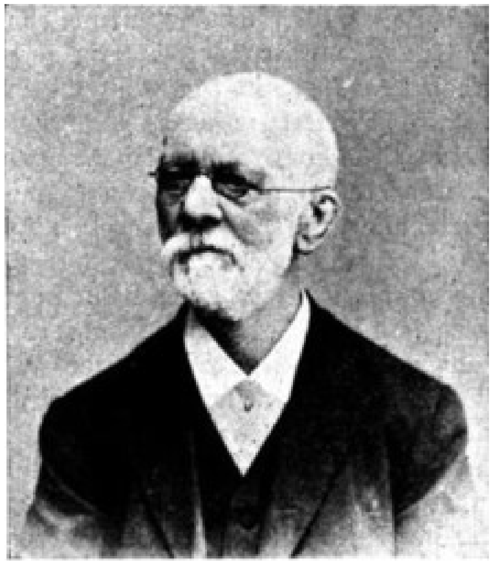
$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}}$$

Equation of the hysteresis loop branch

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}}$$

Parameters of a stress cycle





August Wohler

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min} \quad ,$$ stress range

$$\sigma_a = \frac{\Delta\sigma}{2}$$ stress amplitude

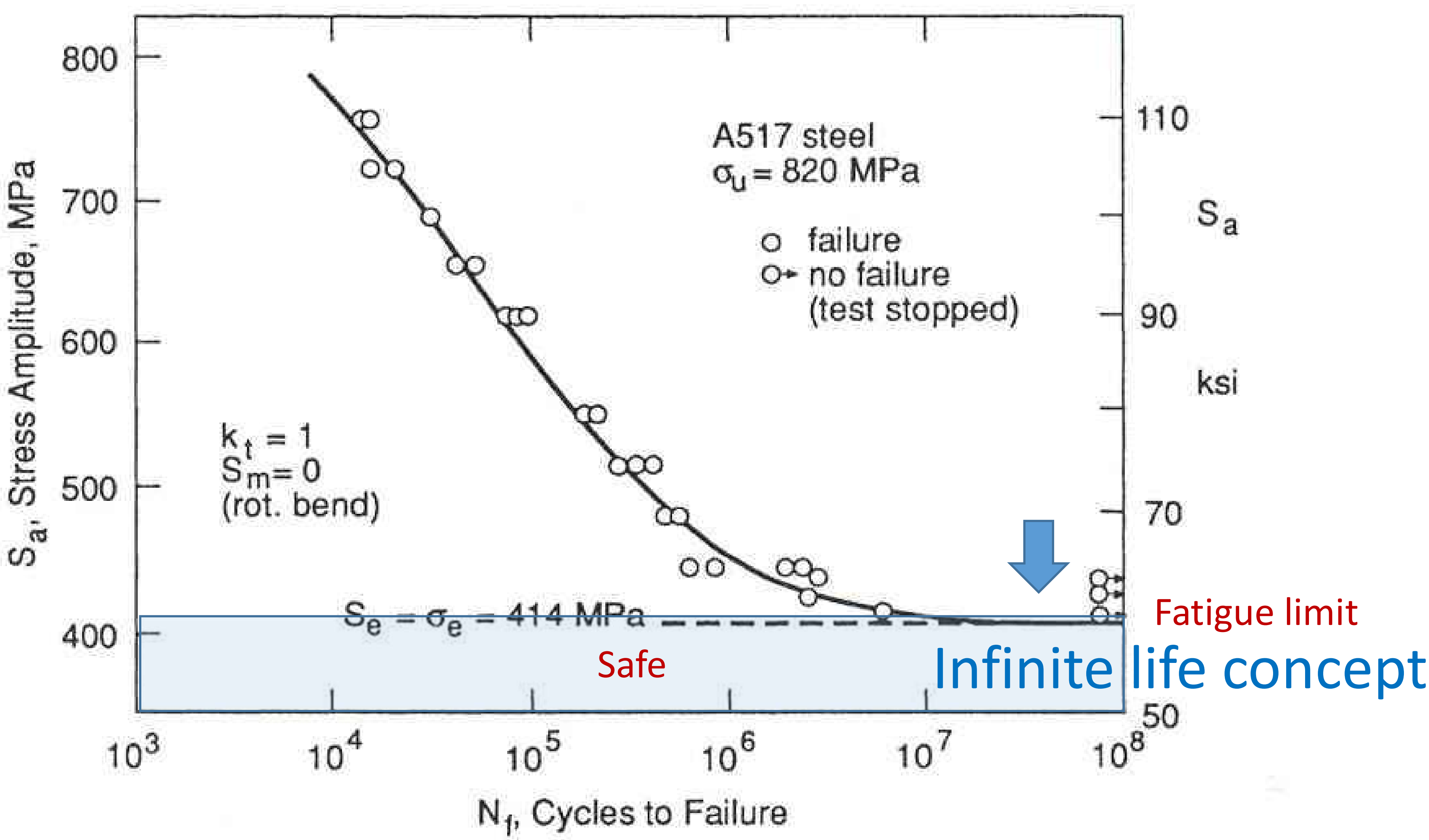
$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$ mean stress

$$\sigma_{\max} = \sigma_m + \sigma_a \quad , \qquad \sigma_{\min} = \sigma_m - \sigma_a$$ min, max stress in a cycle

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \quad , \qquad \Delta\sigma = \sigma_{\max}(1-R)$$ Stress ratio

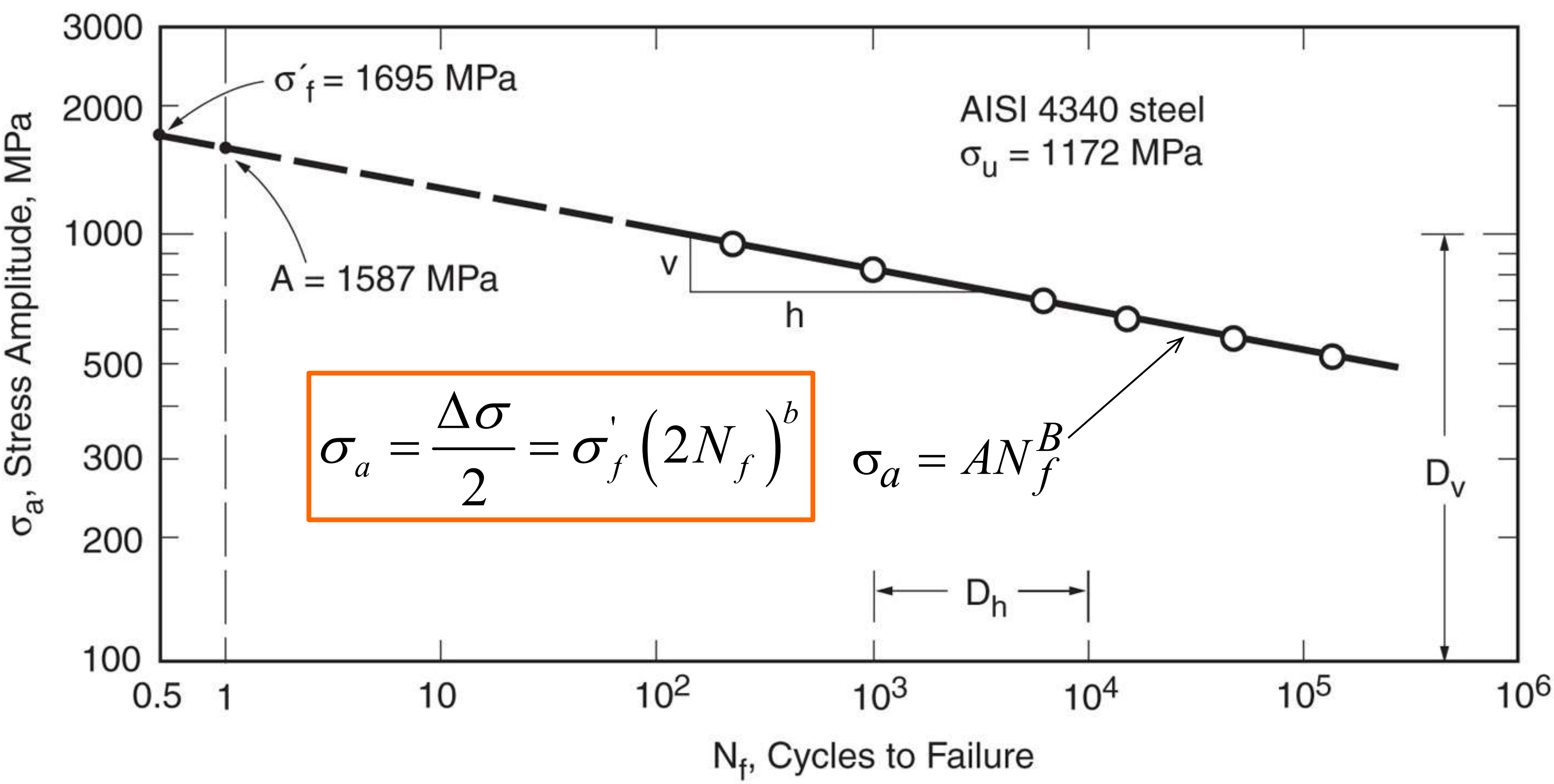
Typical S-N diagram (Wohler curve)

Stress amplitude is kept constant – number of cycles to failure is counted



Power law in fatigue

Power equation fit - straight line on a log-log plot



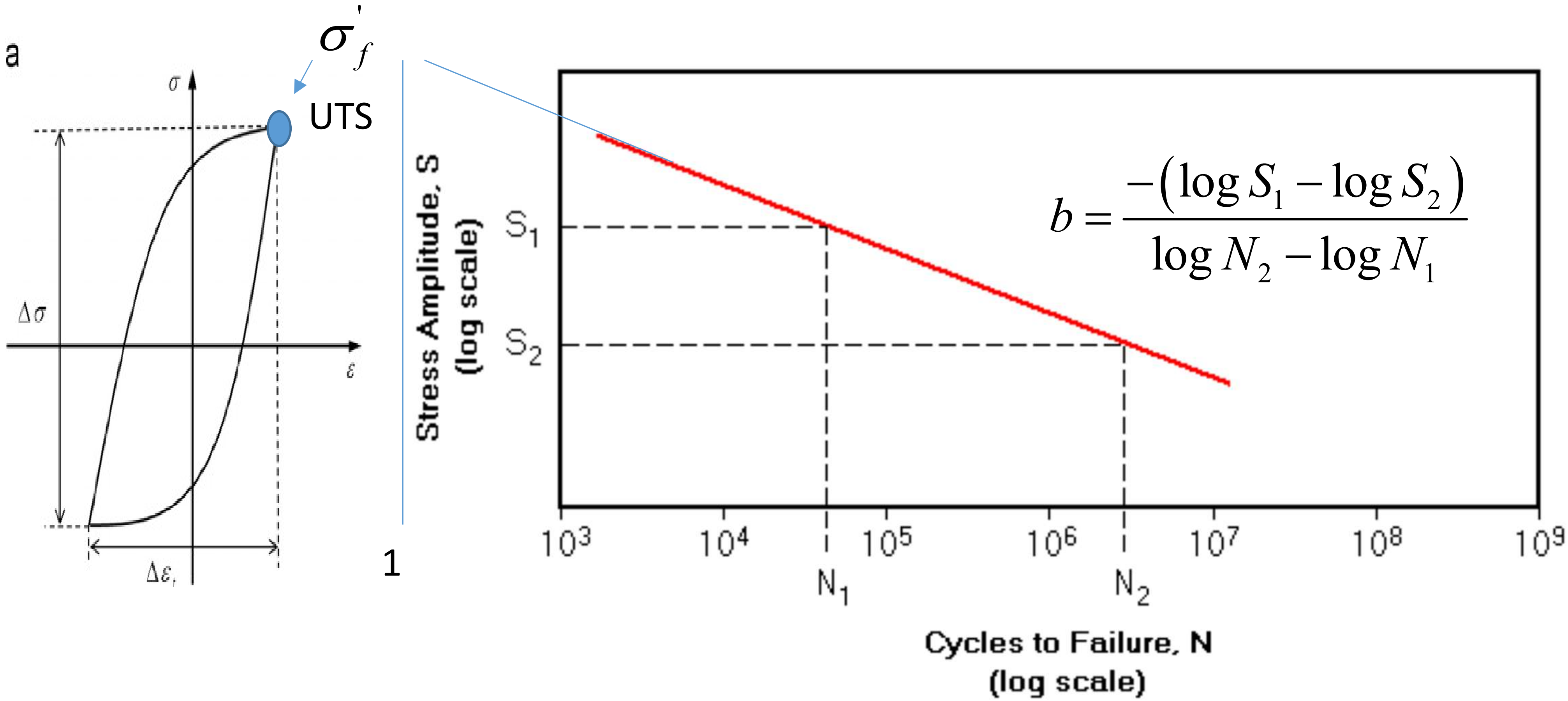
Wohler (S-N) curve equation

- Power law – Basquin equation (empiric)

$$\boxed{\sigma_a = \frac{\Delta\sigma}{2} = \sigma'_f \left(2N_f\right)^b}$$

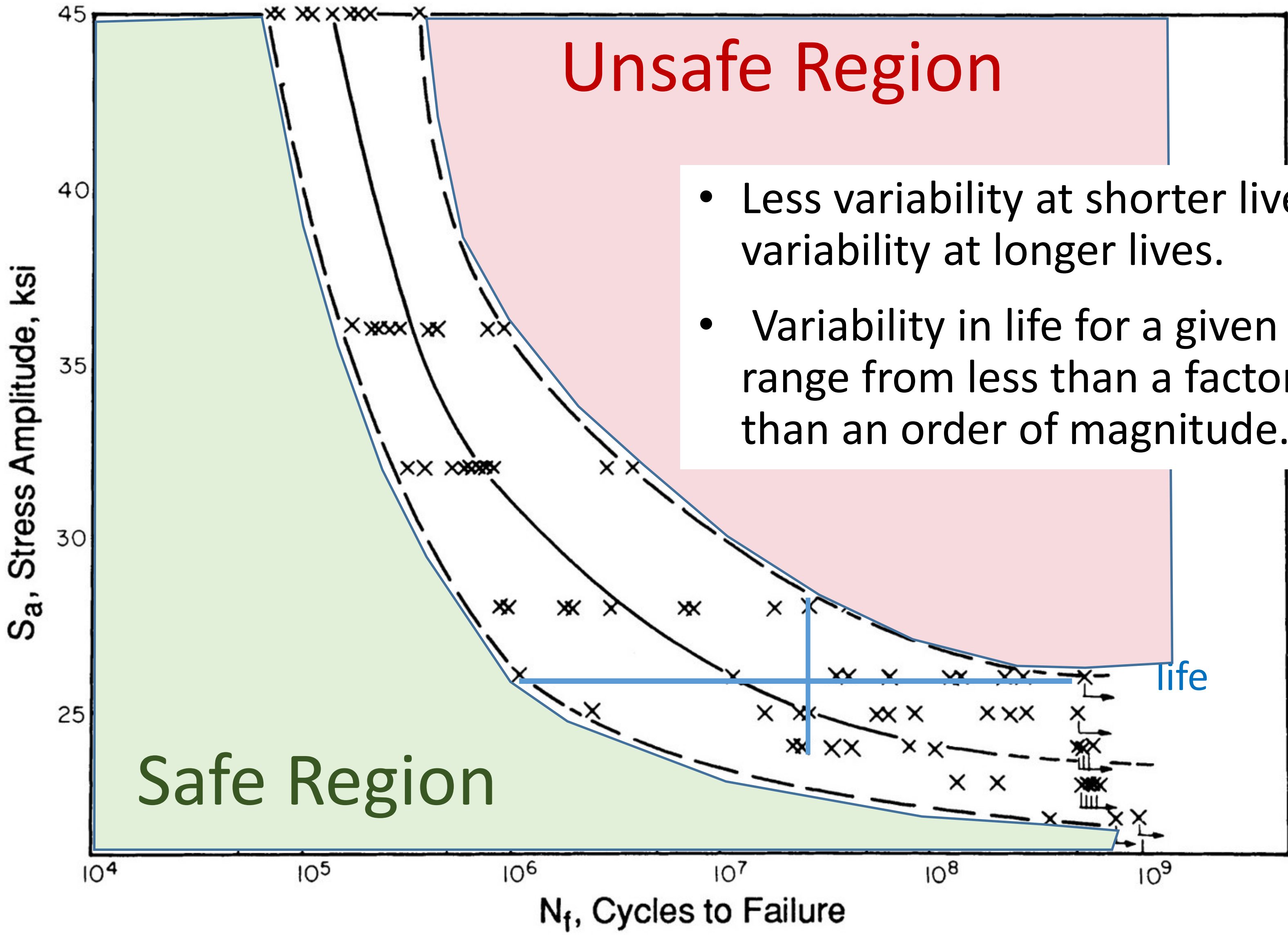
σ'_f - fatigue strength coefficient

b - fatigue strength (Basquin) exponent



$$b = \frac{-(\log S_1 - \log S_2)}{\log N_2 - \log N_1}$$

Statistical scatter of fatigue data

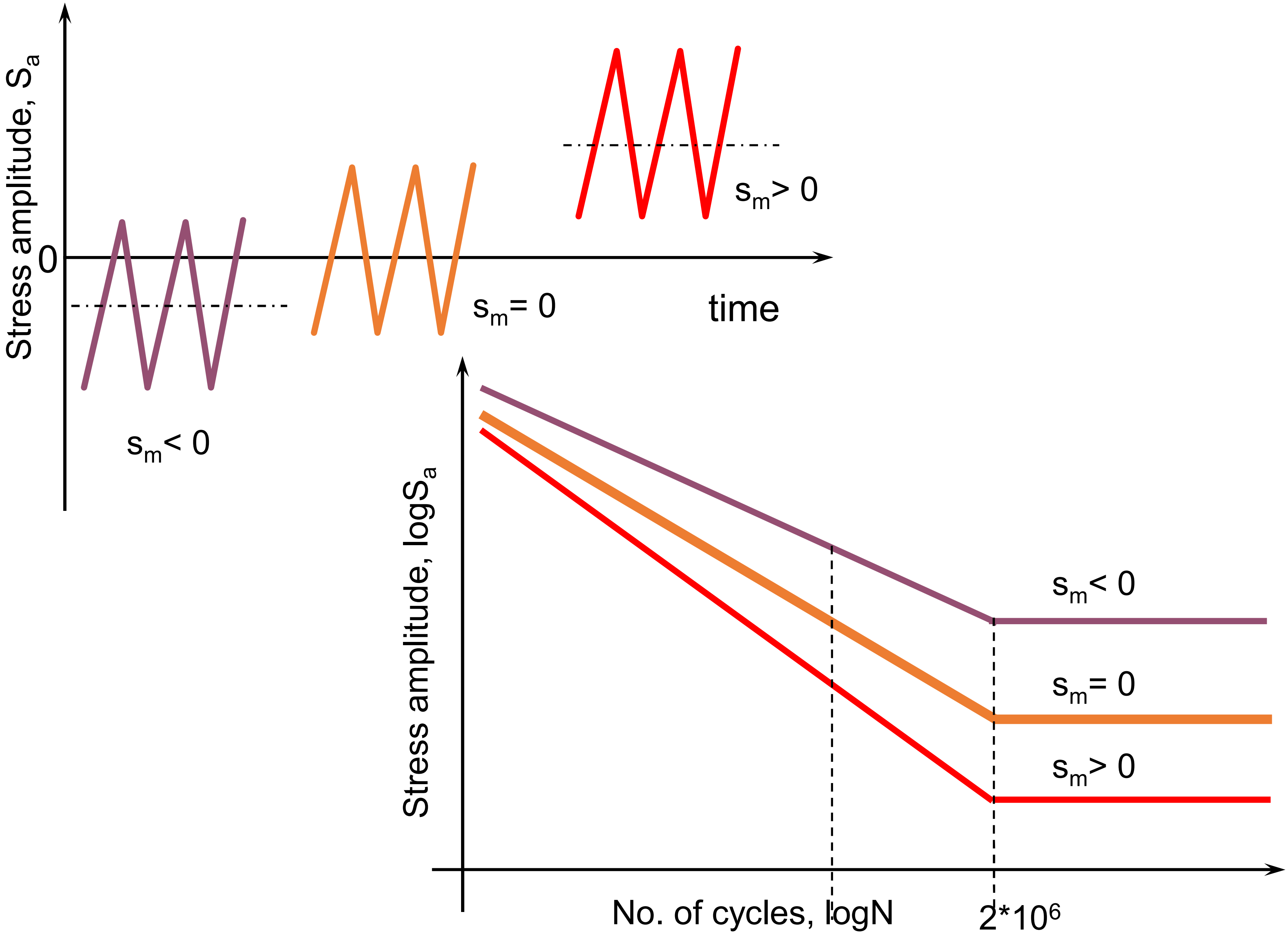


- Less variability at shorter lives and greater variability at longer lives.
- Variability in life for a given stress level can range from less than a factor of two to more than an order of magnitude.

$$\bar{x} \equiv E[x] = \frac{1}{n}\sum_{i=1}^{n} x_i$$ mean

$$\sigma_{std}^2 \equiv \mathrm{var}[x] = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$ variance (dispersion)

Mean Stress Effect



Mean stress correction

$$\sigma_m = 0, R = -1$$

SWT $\quad \sigma_{ar} = \sqrt{\sigma_{max}\sigma_a} = \sigma_{max}\sqrt{\frac{1-R}{2}}$

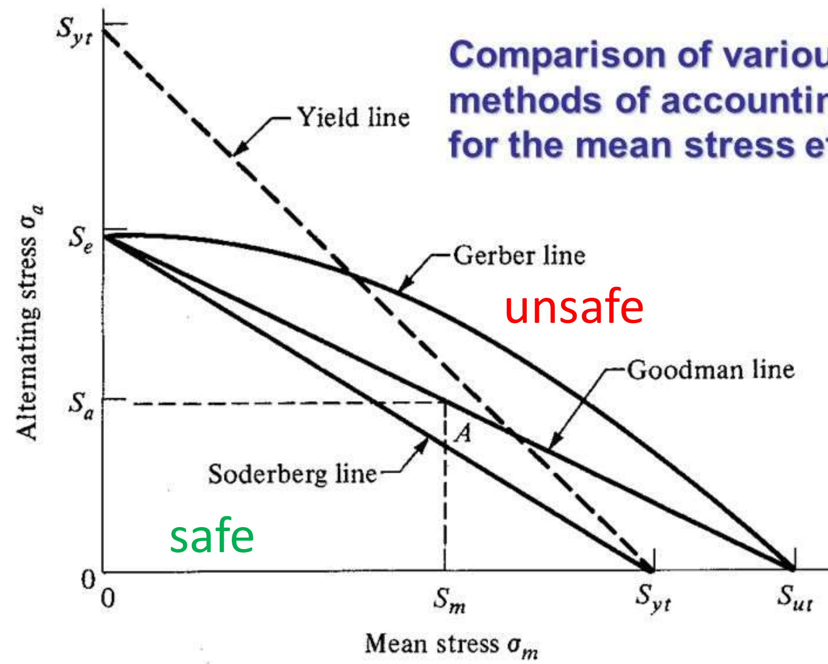
Walker $\quad \sigma_{ar} = \sigma_{max}^{1-\gamma}\sigma_a^{\gamma} = \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma}$

Goodman correction $\quad \sigma_a = \sigma_{ar} \times \left(1 - \frac{\sigma_m}{\sigma_u}\right)$

Soderberg correction $\quad \sigma_a = \sigma_{ar} \times \left(1 - \frac{\sigma_m}{\sigma_y}\right)$

Gerber correction $\quad \sigma_a = \sigma_{ar} \times \left(1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2\right)$

ASME elliptic $\quad \sigma_a = \sigma_{ar} \times \left(1 - \left(\frac{\sigma_m}{\sigma_y}\right)^2\right)$



Comparison of various methods of accounting for the mean stress effect

Most of the experimental data lies between the Goodman and the yield line!

56

Morrow correction $\quad \sigma_a = \sigma_{ar}\left(1 - \frac{\sigma_m}{\tilde{\sigma}_{fB}}\right) \qquad \sigma_a = \sigma_{ar}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)$

$\sigma_f', \quad \tilde{\sigma}_{fB} > \sigma_u$

True fracture stress

Fatigue Limit – Modifying Factors

For many years the emphasis of most fatigue testing was to gain an empirical understanding of the effects of various factors on the base-line S-N curves for ferrous alloys in the intermediate to long life ranges. The variables investigated include:

- Rotational bending fatigue limit, S_e',

- Surface conditions, k_s,

- Size, k_b,

- Mode of loading, k_c,

- Temperature, k_d

- Reliability factor, k_e

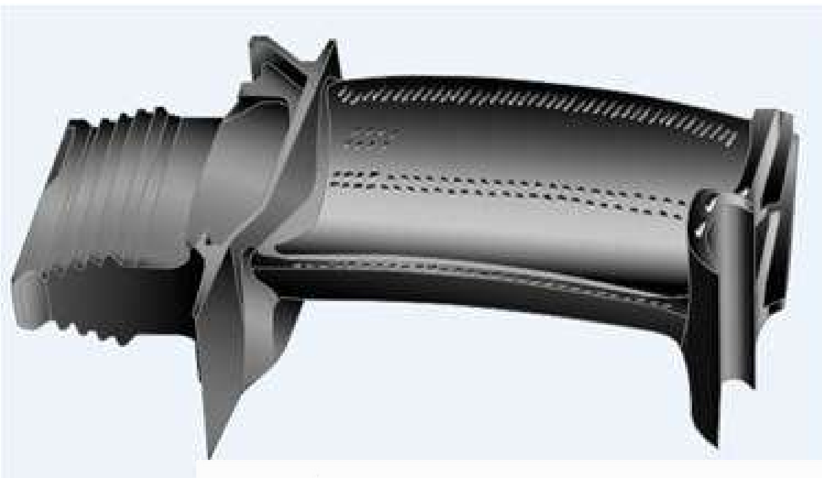
- Miscellaneous effects (notch), k_f

Fatigue limit of a machine part, S_e

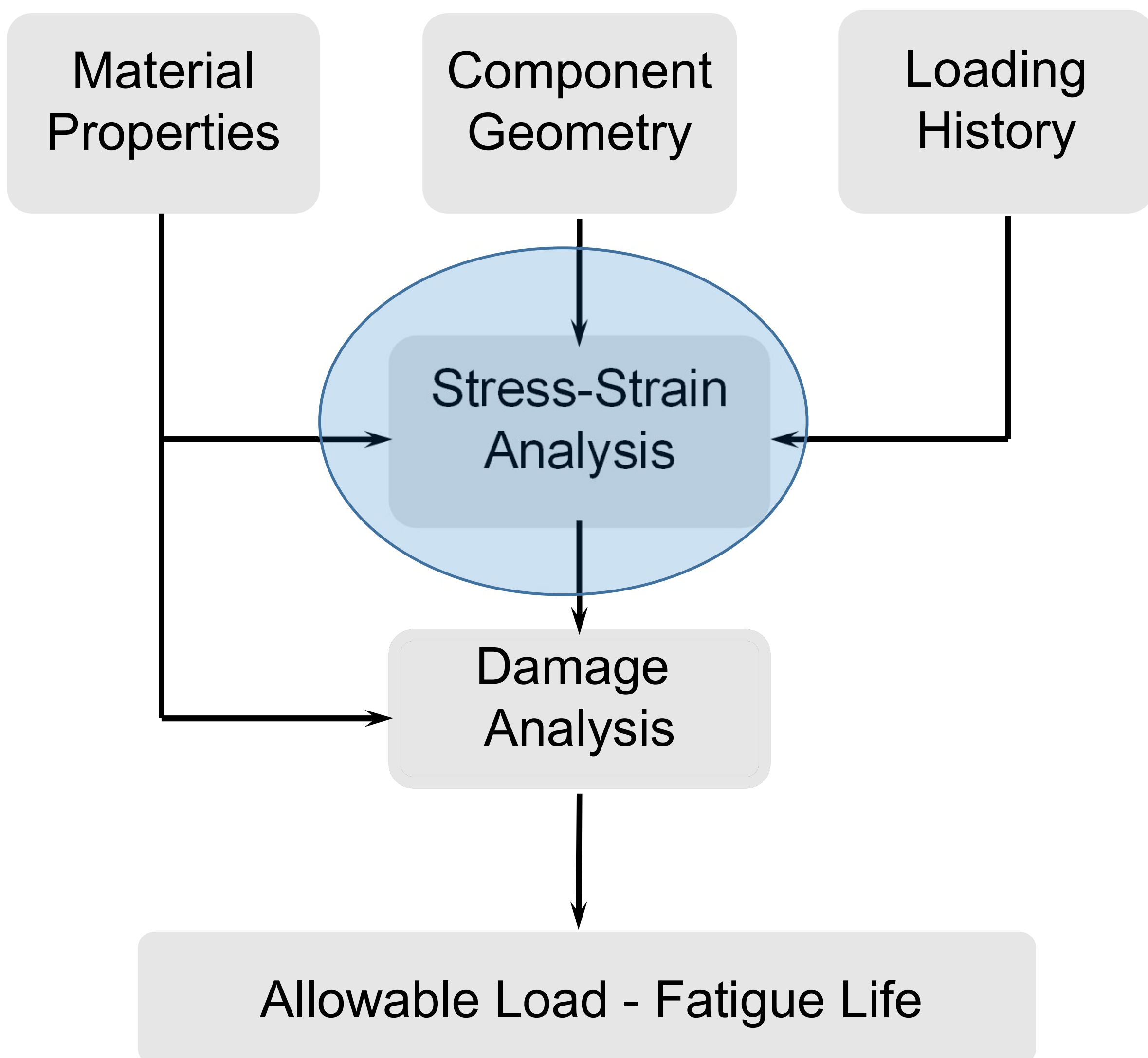
$$S_e = k_s \, k_b \, k_c \, k_d \, k_e \, k_f \cdot S_e'$$

All Above Applies to Smooth Bodies!

Real life assumes more complex shapes



Stress (strain)-based fatigue design



Fracture Mechanics
Pre-requisites

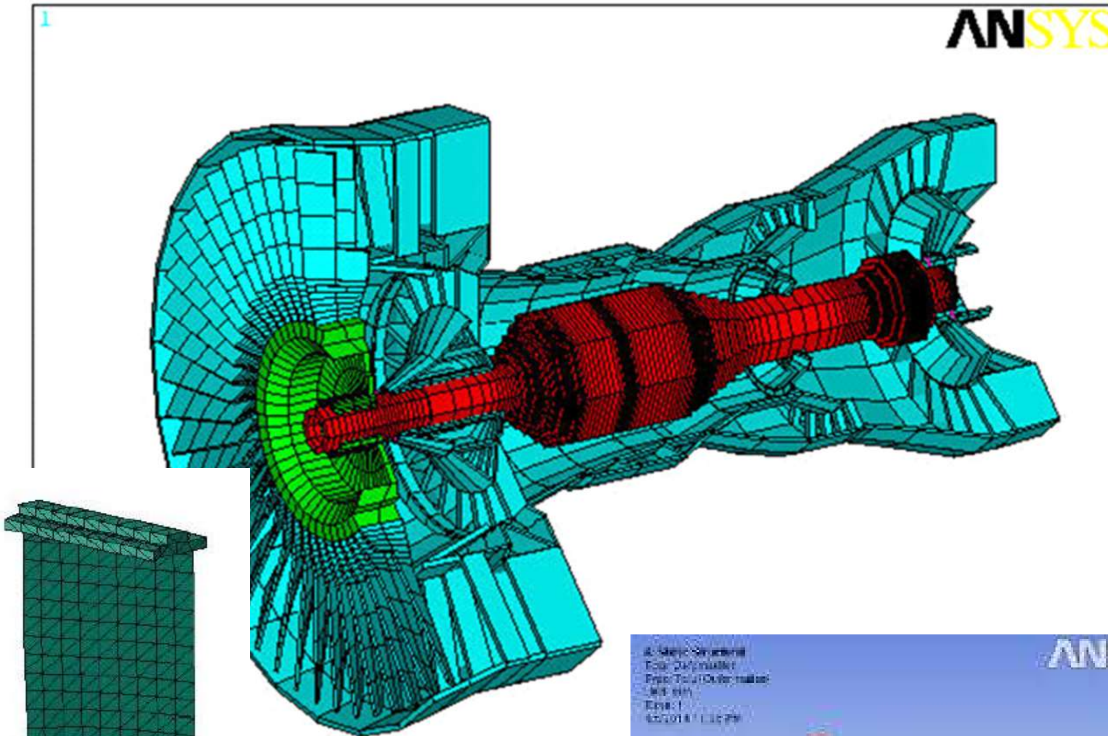
Model it!

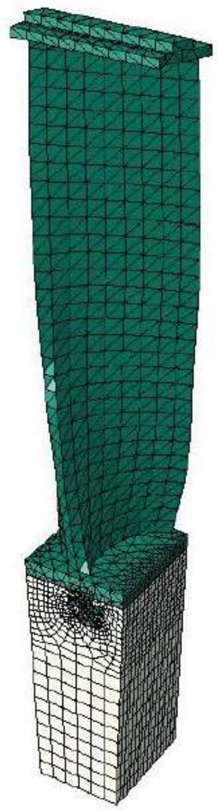
Output

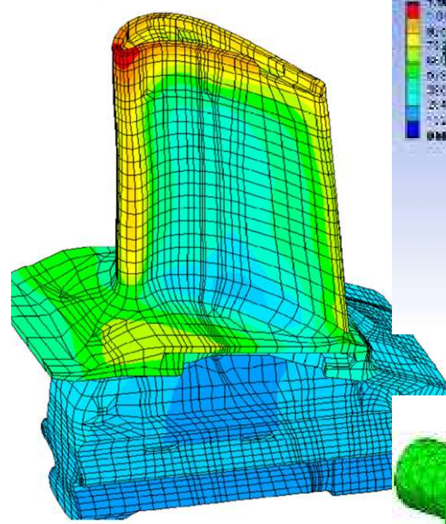
Stress and strain distribution

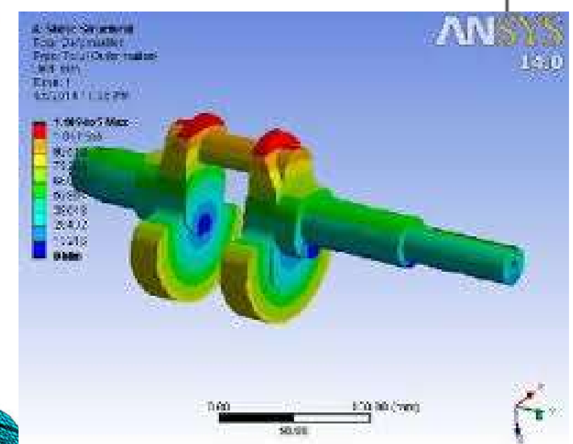
Stress concentration factors

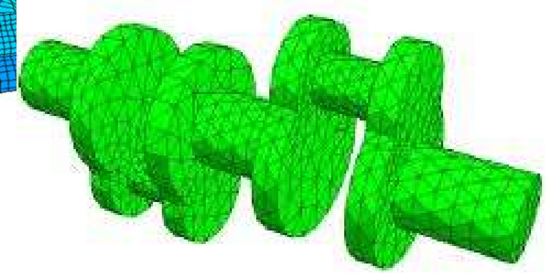
Weakest link

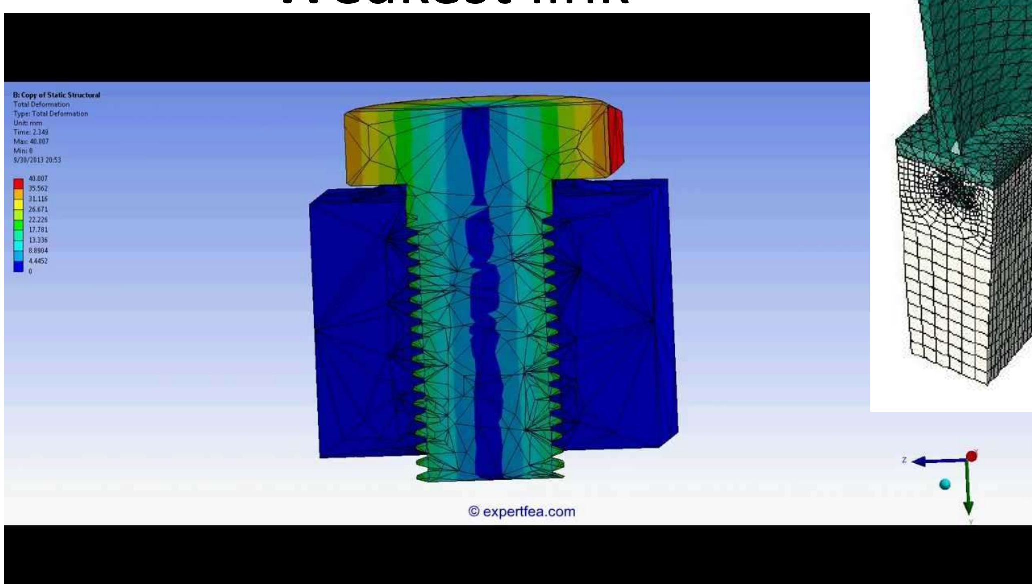




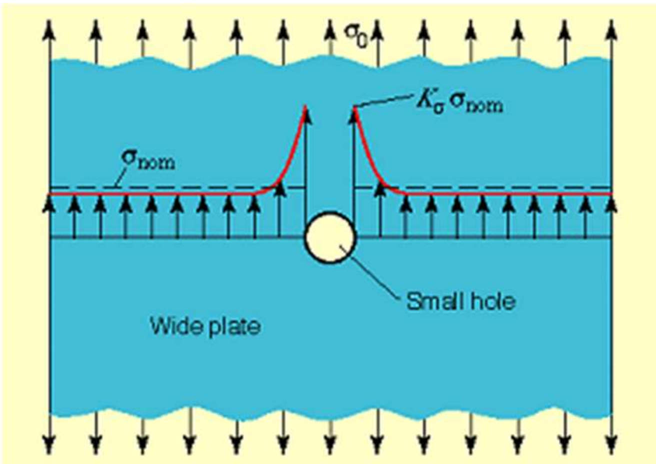


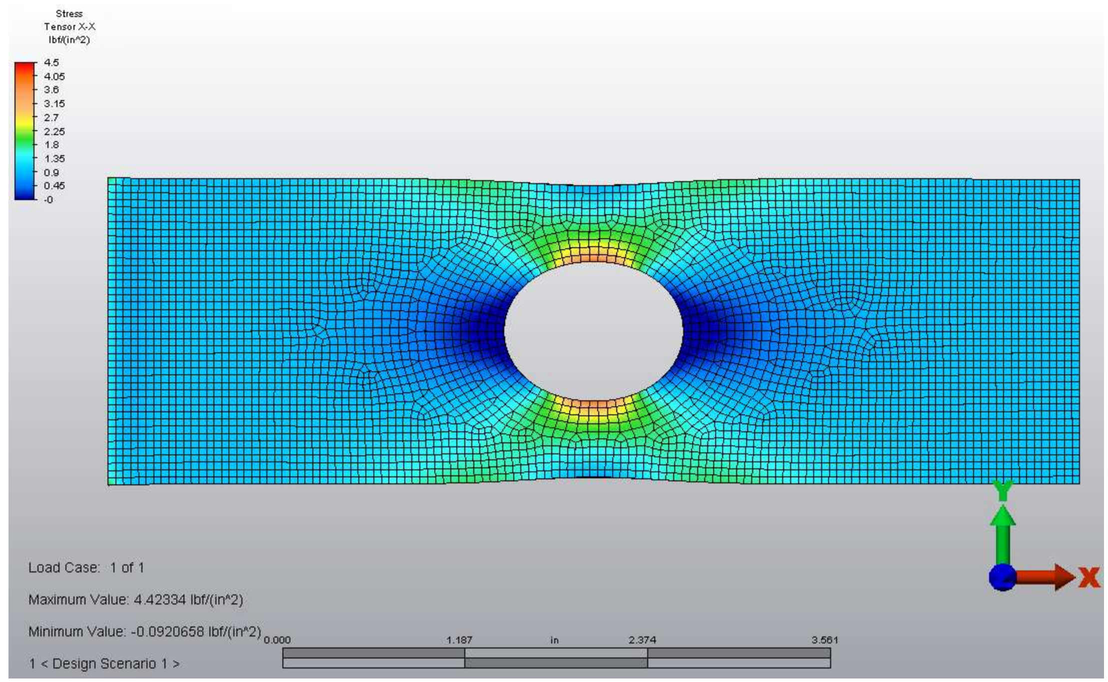


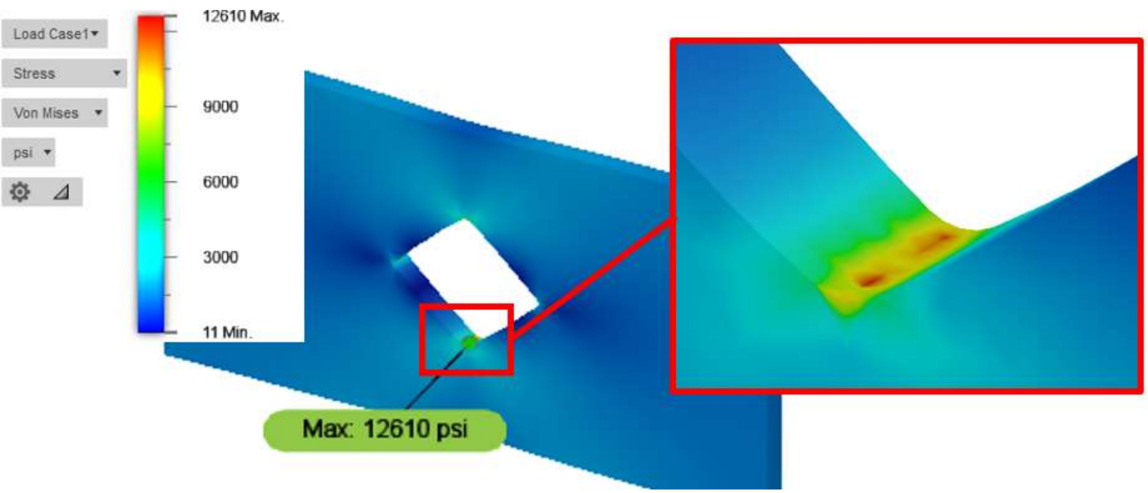


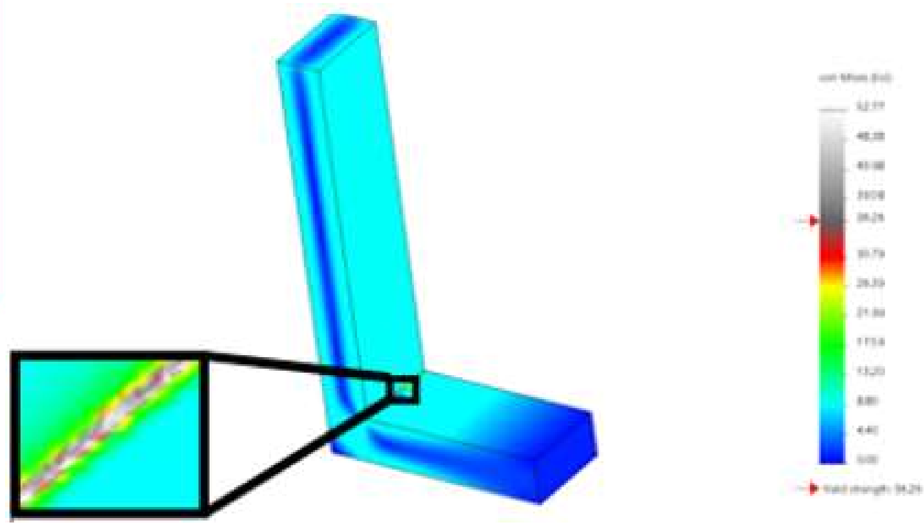


Approximation to Simple Geometries









STRESS AND STRAIN CONCENTRATIONS AND GRADIENTS

- The degree of stress and strain concentration is a factor in the fatigue strength of notched parts.

- It is measured by the **elastic stress concentration factor, k_t**:

$$k_t = \frac{\sigma}{S} = \frac{\varepsilon}{e}$$

As long as $\frac{\sigma}{\varepsilon}$= constant = E

Where:

σ or ε= the maximum stress or strain at the notch

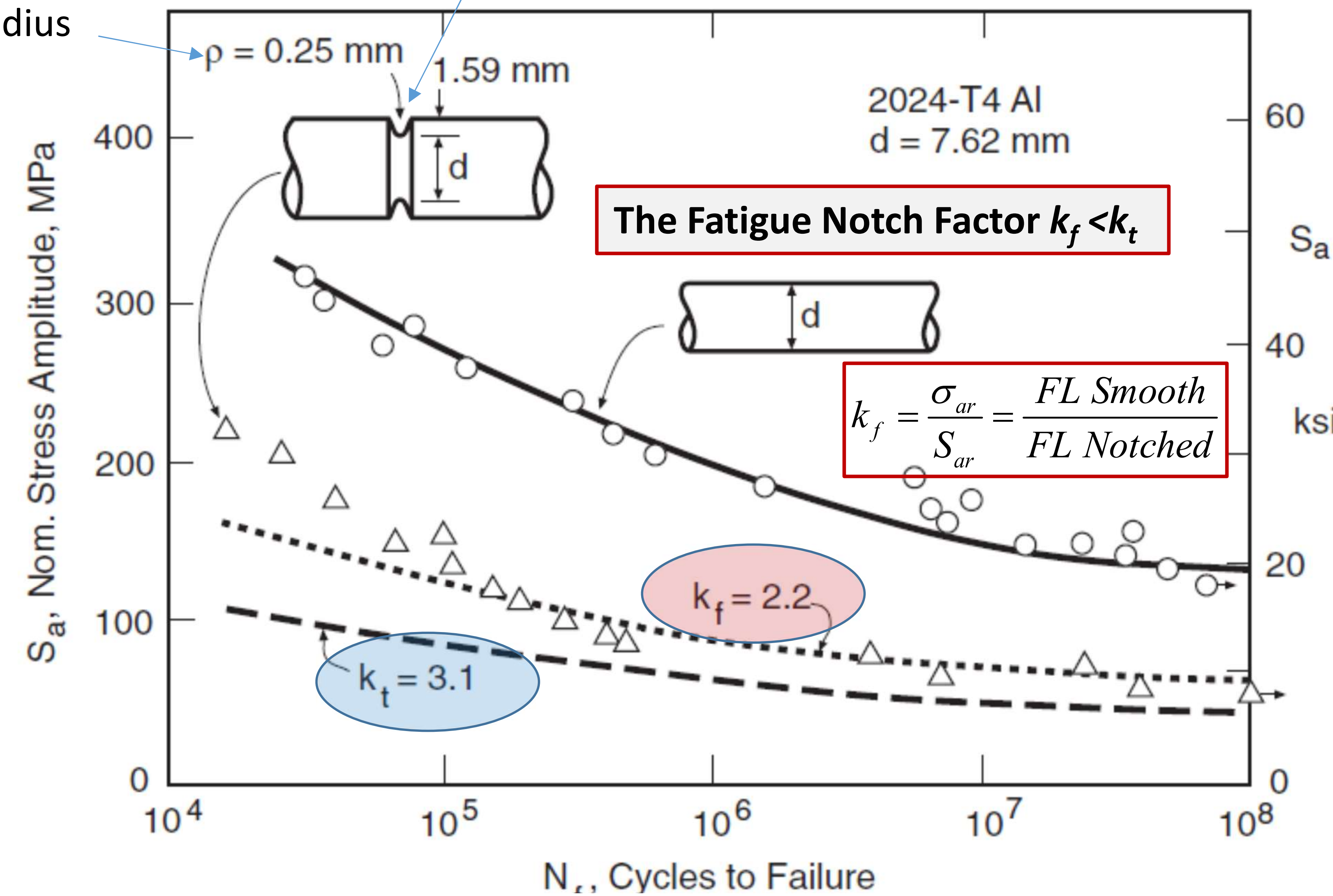
S or *e* = the nominal stress or strain

Fatigue of Notched Members

- k_f vs k_t factors

Stress riser → Stress concentration factor k_t

Radius



S-N APPROACH FOR NOTCHED MEMBERS

(Notch Sensitivity and Fatigue Notch Factor, $\boldsymbol{k_f}$)

- Values of k_f for R= -1 generally range between 1 and k_t, depending on the notch sensitivity of the material, q, which is defined by:

$$\boxed{q = \frac{k_f - 1}{k_t - 1}}$$

- A value of q= 0 (or k_f= 1) indicates no notch sensitivity, whereas a value of q= 1 (or k_f= k_t) indicates full notch sensitivity.
- The fatigue notch factor can then be described through the material notch sensitivity as

$$\boxed{k_f = 1 + q(k_t - 1)}$$

S-NAPPROACH FOR NOTCHED MEMBERS

(Notch Sensitivity and Fatigue Notch Factor, k_f)

- **Peterson** has observed that good approximations for R= -1 loading can also be obtained by using the somewhat similar formula:

$$q = \frac{1}{1+\frac{a}{r}} \qquad \rightarrow \qquad k_f = 1 + \frac{k_t - 1}{1+\frac{a}{r}}$$

where *a* is another material characteristic length.

- An empirical relationship between UTS stress Su and a for steels is given as:

$$a = 0.0254\left(\frac{2070}{S_u}\right)^{1.8}$$ with S_u in MPa and *a* in mm

or

$$a = 0.001\left(\frac{300}{S_u}\right)^{1.8}$$ with S_u in ksi and *a* in inches

- For aluminum alloys, *a* is estimated as 0.635 mm (0.025 in.).

Notch Stresses and Strains

- The relation between and is given by the monotonic stress-strain curve, often represented by the Ramberg-Osgood equation:

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n}$$

- Given nominal elastic stress *S* or strain *e*, the local stress σ and the local strain ε at the notch root can be obtained by:
- experimental methods,
- finite element methods,
- analytical models

Notch stress and strain: Neuber's Rule

- Neuber's rule is the most widely used notch stress/strain model.

$$\boxed{k_\varepsilon k_\sigma = {k_t}^2} \quad \text{or} \quad \varepsilon\sigma = {k_t}^2 \mathrm{eS}$$

local quantities global

- According to this relation, the geometrical mean of the stress and strain concentration factors under plastic deformation conditions remains constant and equal to the theoretical stress concentration factor, k_t.
- This rule agrees with measurements in plane stress situations, such as thin sheets in tension.
- the stress-strain equation is needed: $\varepsilon = \varepsilon_e + \varepsilon_p = \dfrac{\sigma}{E} + \left(\dfrac{\sigma}{K}\right)^{1/n}$

Application of Neuber's Rule

- For nominal elastic behavior, *e*= S/E:

$\varepsilon\sigma = K_t{}^2 \mathrm{e}S$ ➡ Neuber's rule ➡ $\varepsilon\sigma = \frac{(K_t \mathrm{S})^2}{E}$

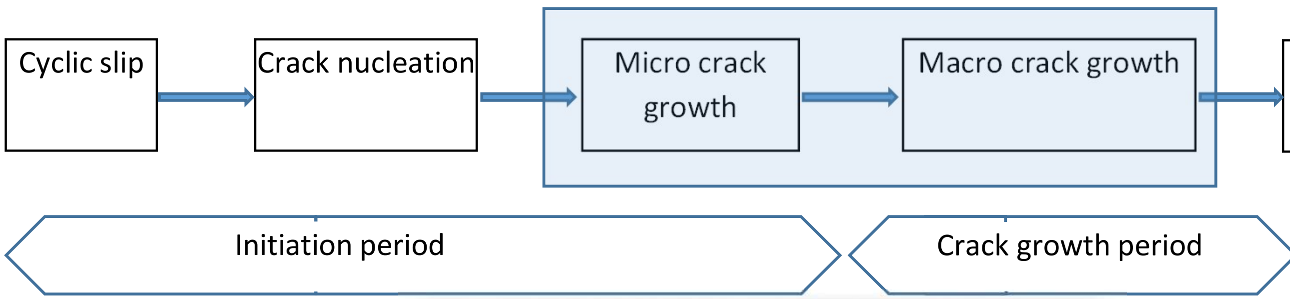
- Combining this equation with the stress-strain equation results in

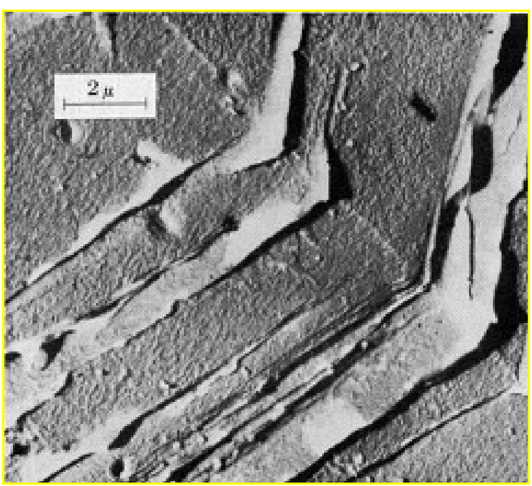
$$\frac{\sigma^2}{E} + \sigma\left(\frac{\sigma}{K}\right)^{1/n} = \frac{(K_t \mathrm{S})^2}{E}$$

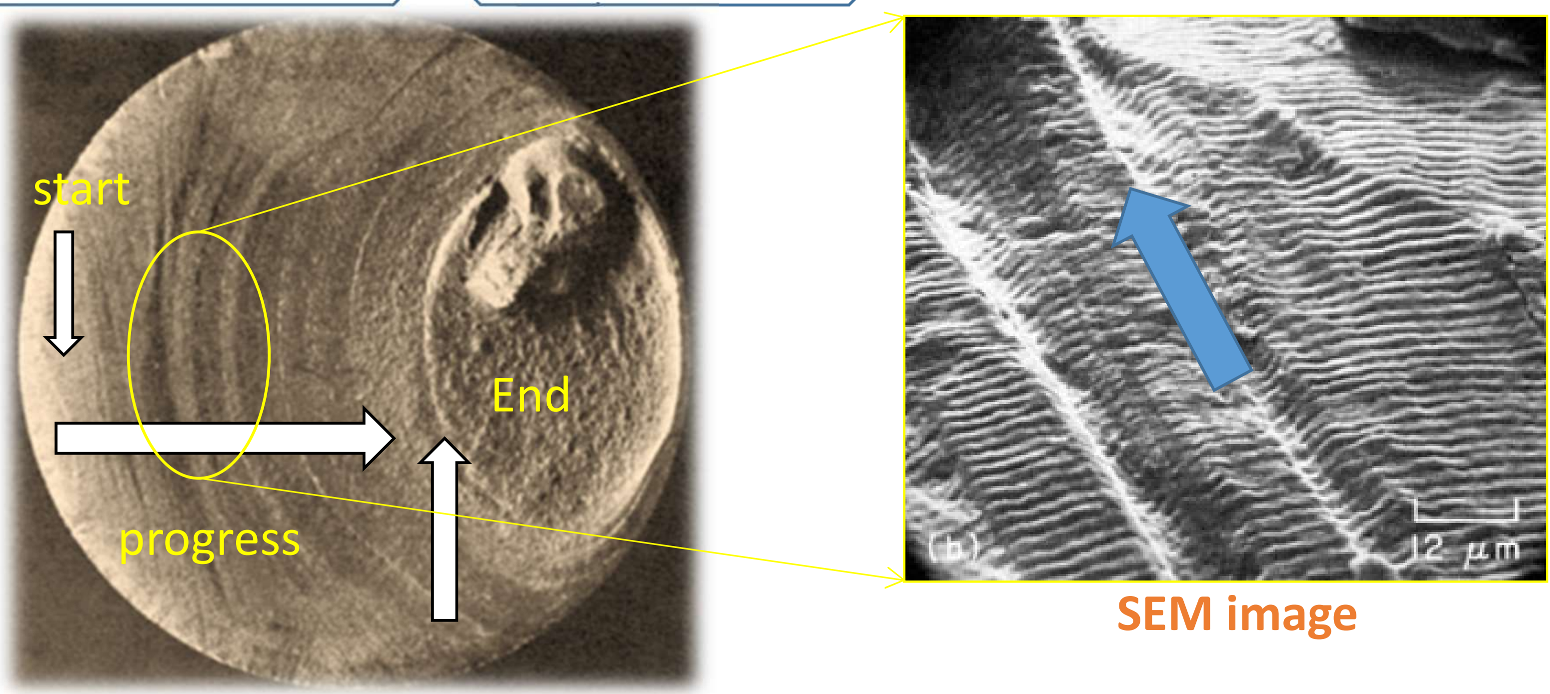
This equation can be solved for notch stress, σ , by iteration or numerical techniques

Stages of Fatigue

- Crack initiation I
- Crack growth II
- Final rupture III



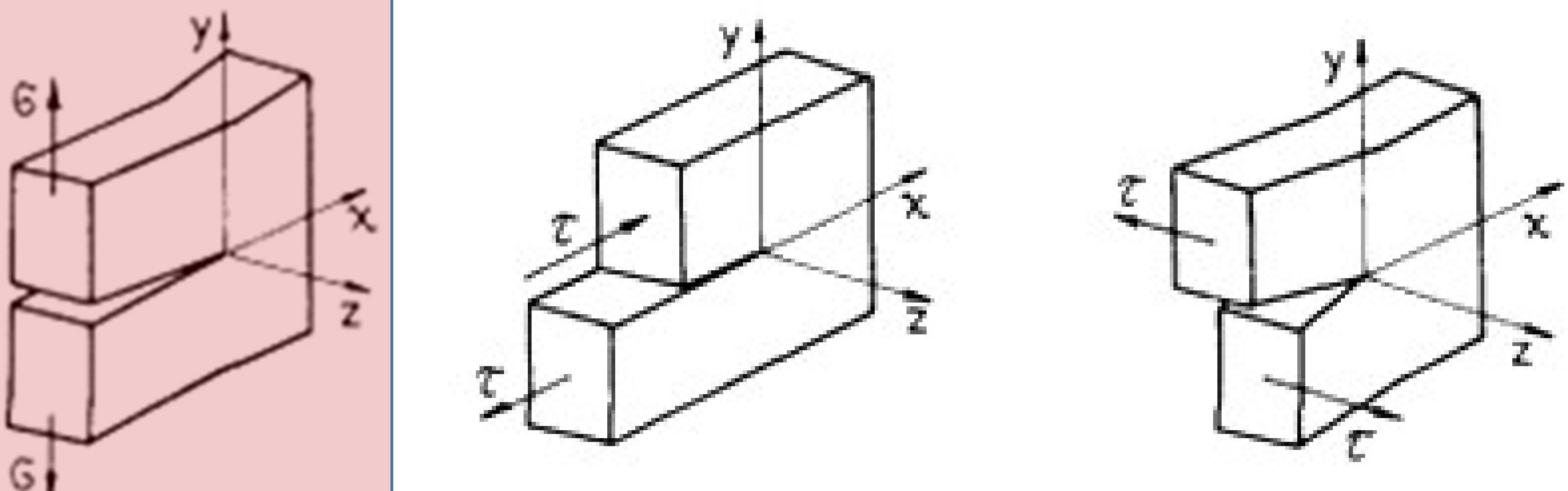




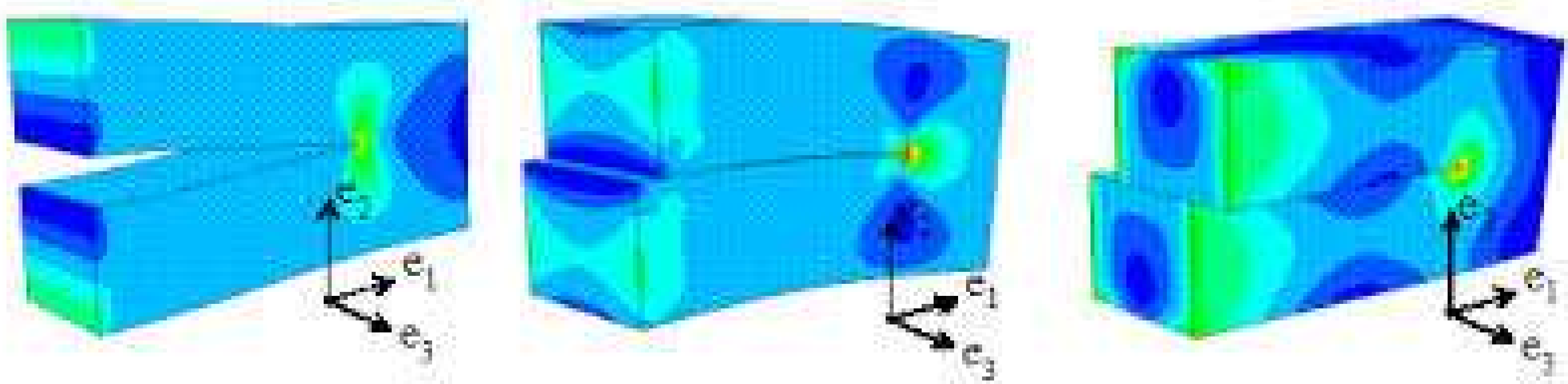
SEM image

Crack modes

I - tensile | II -shear | III -tear



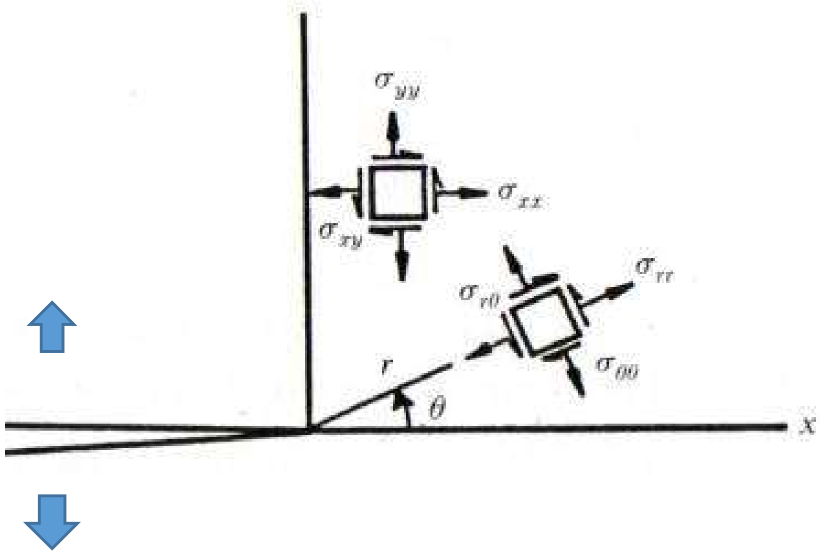
(a) (b) (c)



Mode I | Mode II | Mode III

Crack Tip Stress Solutions

Mode I



$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 - \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 + \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\left(\frac{\theta}{2}\right)\left[\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)\right]$$

$$\text{In general } \sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \quad \text{and} \quad K_I = Y\sigma\sqrt{\pi a}$$

$$\boxed{K = Y\sigma\sqrt{\pi a}}$$

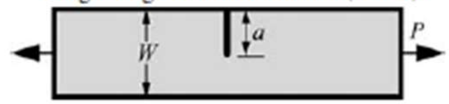
Practical calculation of SIF depending on specimen-crack geometry

$$K_I = Y\sigma\sqrt{\pi a} = \sigma\sqrt{\pi a}\; f\left(\frac{a}{W}\right)$$

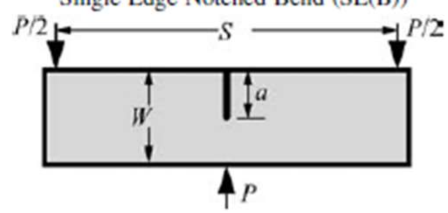
TABLE 2.4
K_I Solutions for Common Test Specimens[a]

GEOMETRY	$f\left(\frac{a}{W}\right)^*$
Single Edge Notched Tension (SENT)	$\frac{\sqrt{2\tan\frac{\pi a}{2W}}}{\cos\frac{\pi a}{2W}}\left[0.752+2.02\left(\frac{a}{W}\right)+0.37\left(1-\sin\frac{\pi a}{2W}\right)^3\right]$
Single Edge Notched Bend (SE(B))	$\frac{3\frac{S}{W}\sqrt{\frac{a}{W}}}{2\left(1+2\frac{a}{W}\right)\left(1-\frac{a}{W}\right)^{3/2}}\left[1.99-\frac{a}{W}\left(1-\frac{a}{W}\right)\left\{2.15-3.93\left(\frac{a}{W}\right)+2.7\left(\frac{a}{W}\right)^2\right\}\right]$
Center Cracked Tension (CCT)	$\sqrt{\frac{\pi a}{4W}\sec\left(\frac{\pi a}{2W}\right)}\left[1-0.025\left(\frac{a}{W}\right)^2+0.06\left(\frac{a}{W}\right)^4\right]$
Double Edge Notched Tension (DENT)	$\frac{\sqrt{\frac{\pi a}{2W}}}{\sqrt{1-\frac{a}{W}}}\left[1.122-0.561\left(\frac{a}{W}\right)-0.205\left(\frac{a}{W}\right)^2+0.471\left(\frac{a}{W}\right)^3+0.190\left(\frac{a}{W}\right)^4\right]$
Compact Specimen	$\frac{2+\frac{a}{W}}{\left(1-\frac{a}{W}\right)^{3/2}}\left[0.886+4.64\left(\frac{a}{W}\right)-13.32\left(\frac{a}{W}\right)^2+14.72\left(\frac{a}{W}\right)^3-5.60\left(\frac{a}{W}\right)^4\right]$

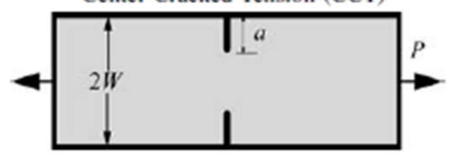
Single Edge Notched Tension (SENT)



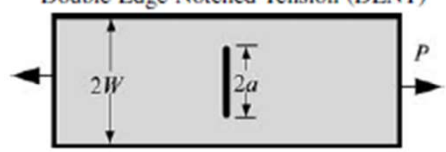
Single Edge Notched Bend (SE(B))



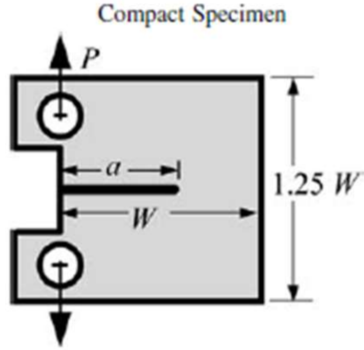
Center Cracked Tension (CCT)



Double Edge Notched Tension (DENT)

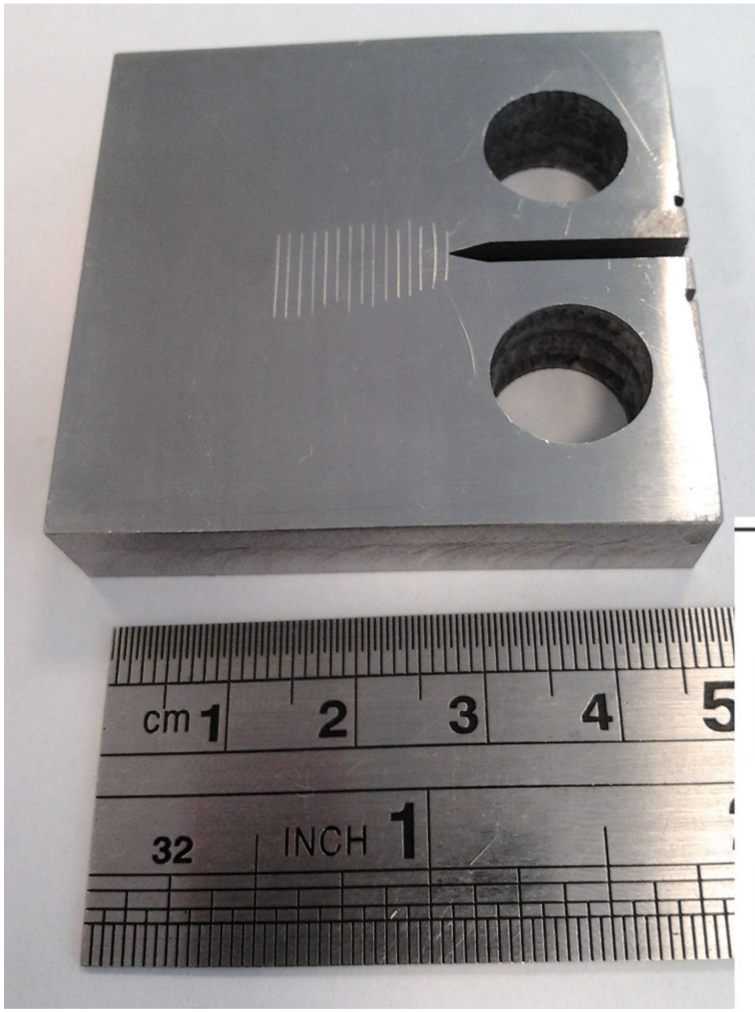


Compact Specimen

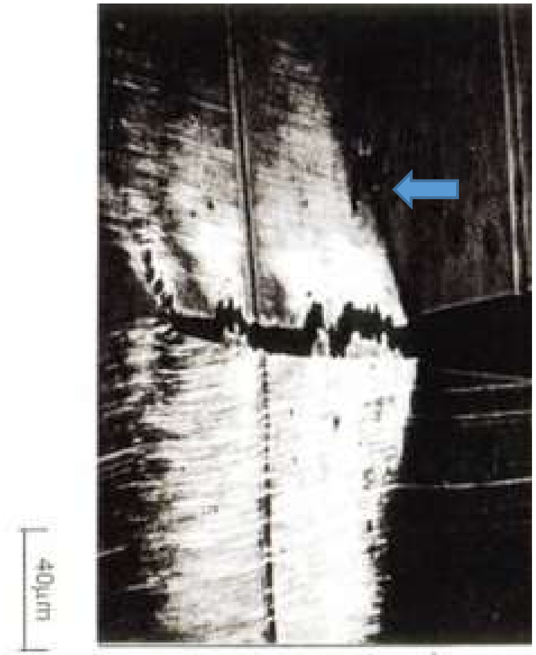


Fatigue crack growth testing

Compact tension (CT) specimen

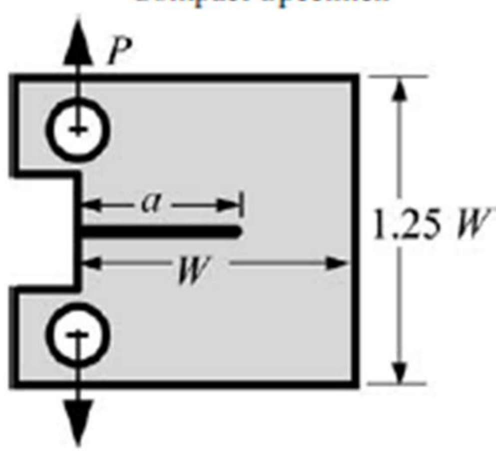


$$K_I = \sigma\sqrt{\pi a}\; f\left(\frac{a}{W}\right)$$



Observe crack growth from the notch

Compact Specimen



$$\frac{2+\frac{a}{W}}{\left(1-\frac{a}{W}\right)^{3/2}}\left[0.886+4.64\left(\frac{a}{W}\right)-13.32\left(\frac{a}{W}\right)^2 +14.72\left(\frac{a}{W}\right)^3-5.60\left(\frac{a}{W}\right)^4\right]$$

Stages of fatigue crack growth

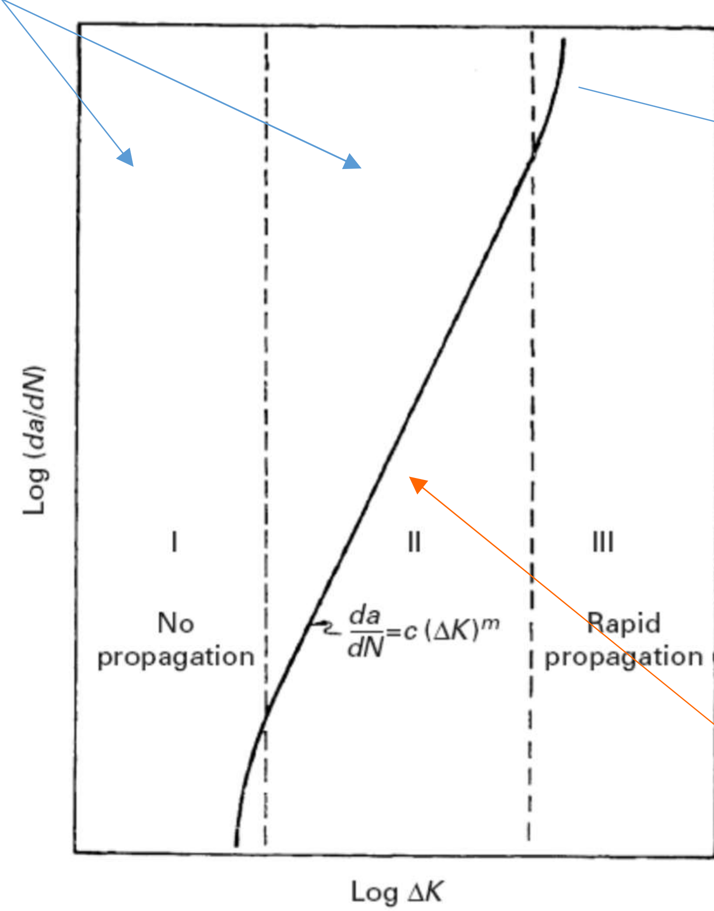
$$r_c = \frac{1}{3\pi}\left(\frac{\Delta K_I}{2\sigma'_{YS}}\right)^2$$

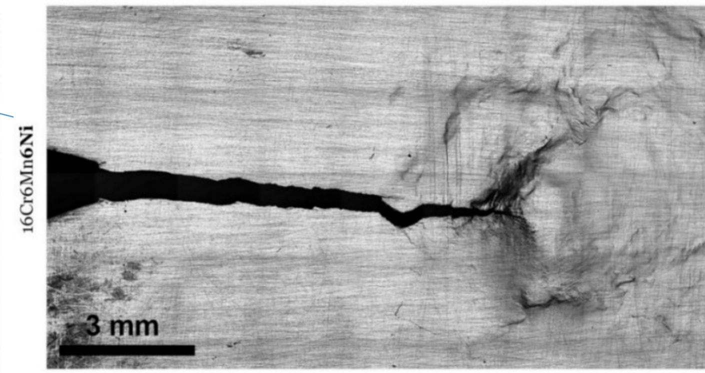
$$r_p = \frac{1}{\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

monotonic

Plastic zone is small

Plastic zone is large





For small ΔK (region I), crack propagation is **difficult to predict** since it depends on **microstructure** and **flow properties** of the material.
Here, the growth may even come to an **arrest.**

Stable crack growth

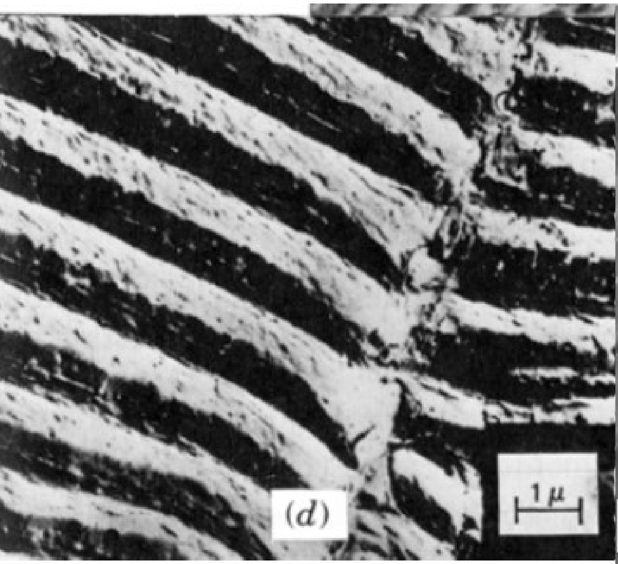
$$\frac{da}{dN} = C(\Delta K)^m$$

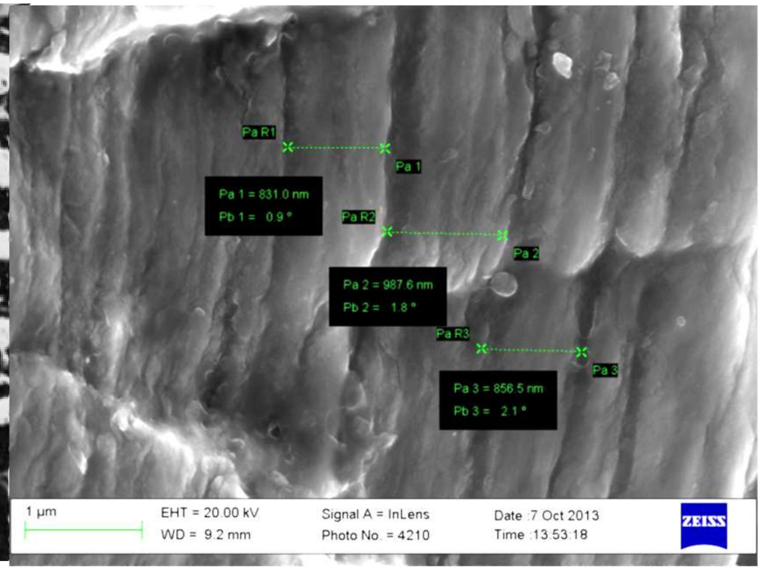
Paris –Erdogan Law

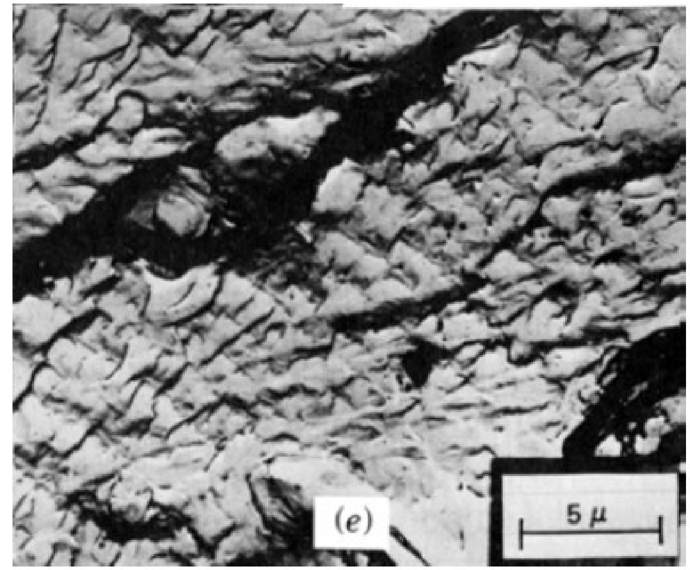
$$\Delta K = Y\,\Delta\sigma\sqrt{\pi a}$$

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$

Stable crack growth: Fatigue striations







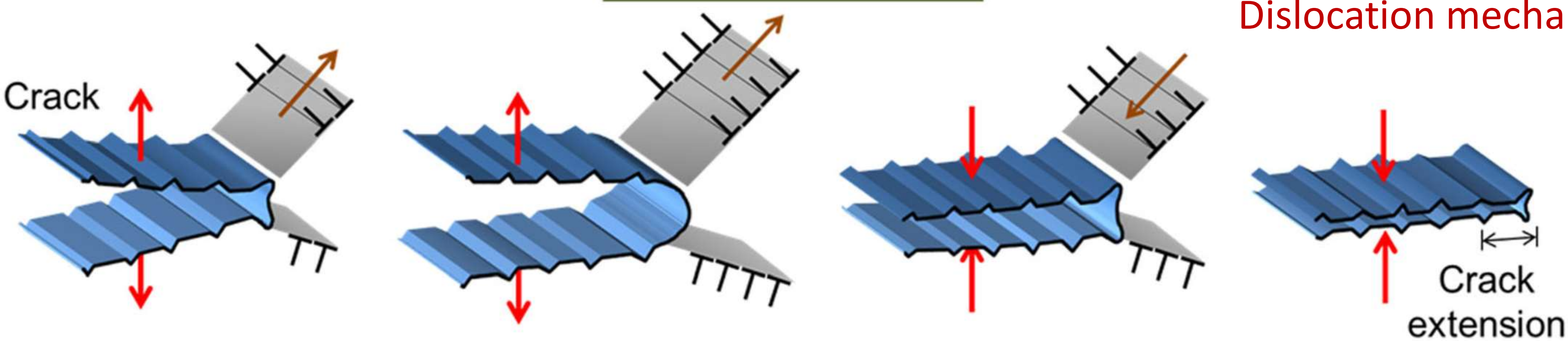
"Ductile" striations, with large, regular size and spacing

"Brittle" striations, where successive crack front locations are still marked

Δa ➡ $\Delta a / \Delta t$ Crack growth velocity

Plastic blunting model

Dislocation mechanism



Life Calculations

To calculate the life for crack growth, an integration based on the crack growth rate curve is needed between initial and final crack sizes.

$$\frac{da}{dN} = f(\Delta K, R)\,, \qquad N_f = \int_{a_i}^{a_f} \frac{da}{f(\Delta K, R)}$$

Although closed form integration is possible in some cases, numerical integration or an equivalent iterative summing procedure is often needed.

$$\Delta K = Y\,\Delta\sigma\sqrt{\pi a} \qquad (Y \approx \text{ constant})$$

$$\frac{da}{dN} = C(\Delta K)^m$$

If the geometry function *Y* is approximately constant, and for power-law behavior, a closed-form equation results.

Closed-form equation

$$N_f = \int dN = \int_{a_i}^{a_f} \frac{da}{CY^m \Delta\sigma^m \pi^{m/2} a^{m/2}} = \frac{1}{C\Delta\sigma^m \pi^{m/2}} \int_{a_i}^{a_f} \frac{da}{Y^m(a) a^{m/2}} = \frac{1}{CY^m \Delta\sigma^m \pi^{m/2}} \int_{a_i}^{a_f} \frac{da}{a^{m/2}}$$

- is not correct if Y significantly changes with a between the limits a_i and a_f .

$$N_f = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C(Y \Delta\sigma \sqrt{\pi})^m (1-m/2)} \qquad (m \neq 2)$$

$$a_f = \frac{1}{\pi}\left(\frac{K_c}{Y\sigma_{\max}}\right)^2$$

Can be obtained from FCG curve or calculated using K_c (K_{Ic}) fracture toughness data

$$(m=2) \qquad \frac{da}{dN} = C(\Delta K)^2 \qquad \frac{da}{dN} \propto a \qquad \frac{da}{dN} \propto (\Delta a)$$

$$N_f = \frac{1}{\pi C Y^2 \Delta\sigma^2} \int_{a_i}^{a_f} \frac{da}{a} = \frac{1}{\pi C Y^2 \Delta\sigma^2} \ln\left(\frac{a_f}{a_i}\right)$$

$$a(N) = N_0 \exp\left(\pi C Y^2 \Delta\sigma^2 N\right)$$

$$\boxed{\frac{da}{dN} \propto (\Delta a) \approx \delta_t = \beta \frac{(\Delta K)^2}{\sigma_y' E'}}$$

Advantages of models based on CTOD:

1. Physical justification including dislocation-based modelling

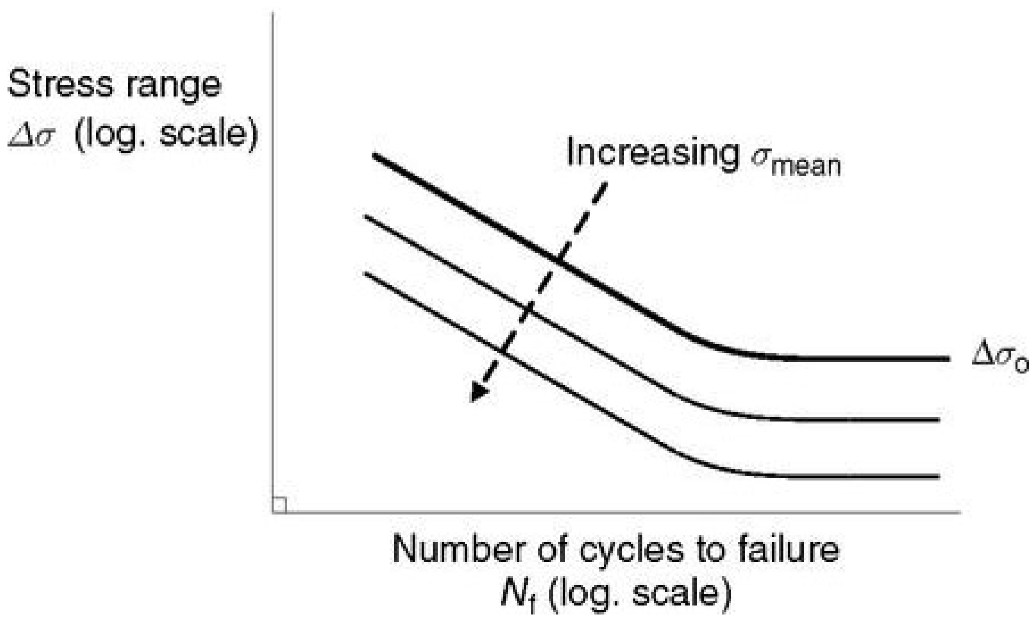
2. Application to multiaxial fatigue

$$(m=4) \qquad N_f = \frac{1}{C(Y\Delta\sigma\sqrt{\pi})^m (m/2-1)} \left(\frac{1}{a_i^{m/2-1}} - \frac{1}{a_f^{m/2-1}}\right) =$$

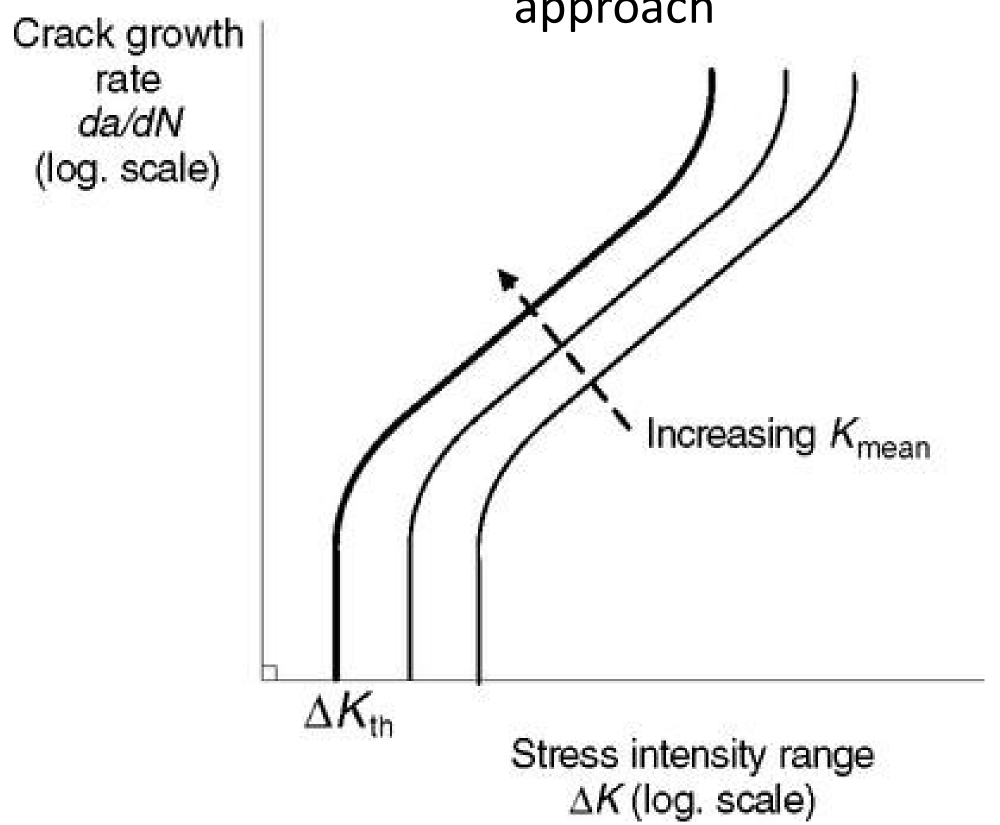
$$\frac{1}{C Y^4 \pi^2 \Delta\sigma^4} \left(\frac{1}{a_i} - \frac{1}{a_f}\right)$$

Effect of mean stress (intensity factor)

Stress-based approach



Defect tolerant – based (crack growth) approach



Forman equation

A commonly used equation depicting mean stress effects in regions II and III is the Forman equation:

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_c - \Delta K} = \frac{C(\Delta K)^m}{(1-R)\left(K_c - K_{\max}\right)}$$

C and m are empirical material fatigue crack growth rate constants and K_c is fracture toughness of the material.

The Forman equation is a modification of the Paris equation to incorporate mean stress and region III fatigue crack growth behavior.

As K_{max} approaches K_c, the denominator approaches zero, thus the crack growth rate, da/dN , gets very large. This describes region III crack growth.

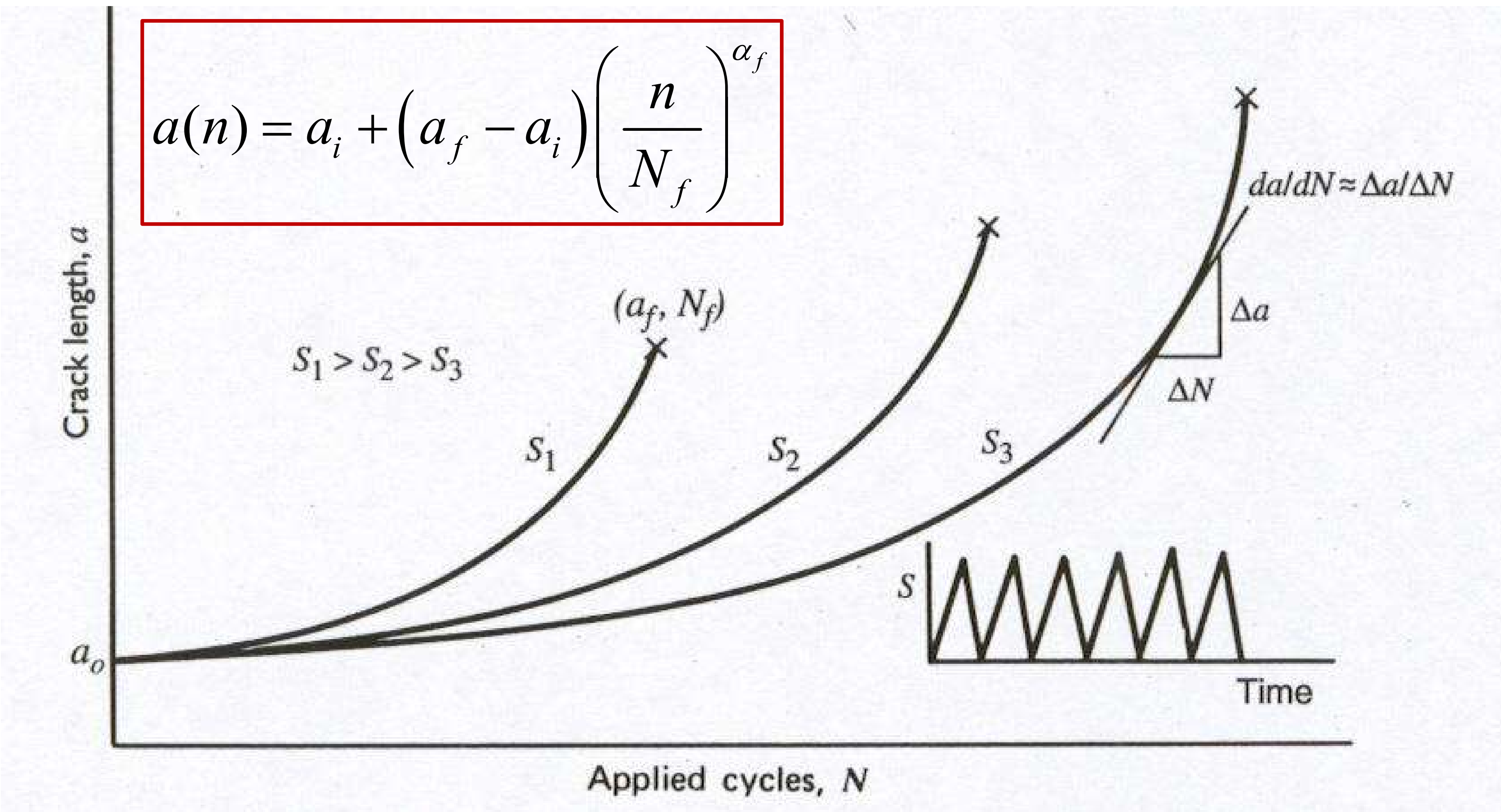
Other forms of correction are also possible

$$\frac{da}{dN} = C_2\left(\frac{\Delta K}{(1-R)^{1-\gamma_2}}\right)^{m_2} \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{\Delta K}{(1-R)K_c}\right)^q}$$

Damage

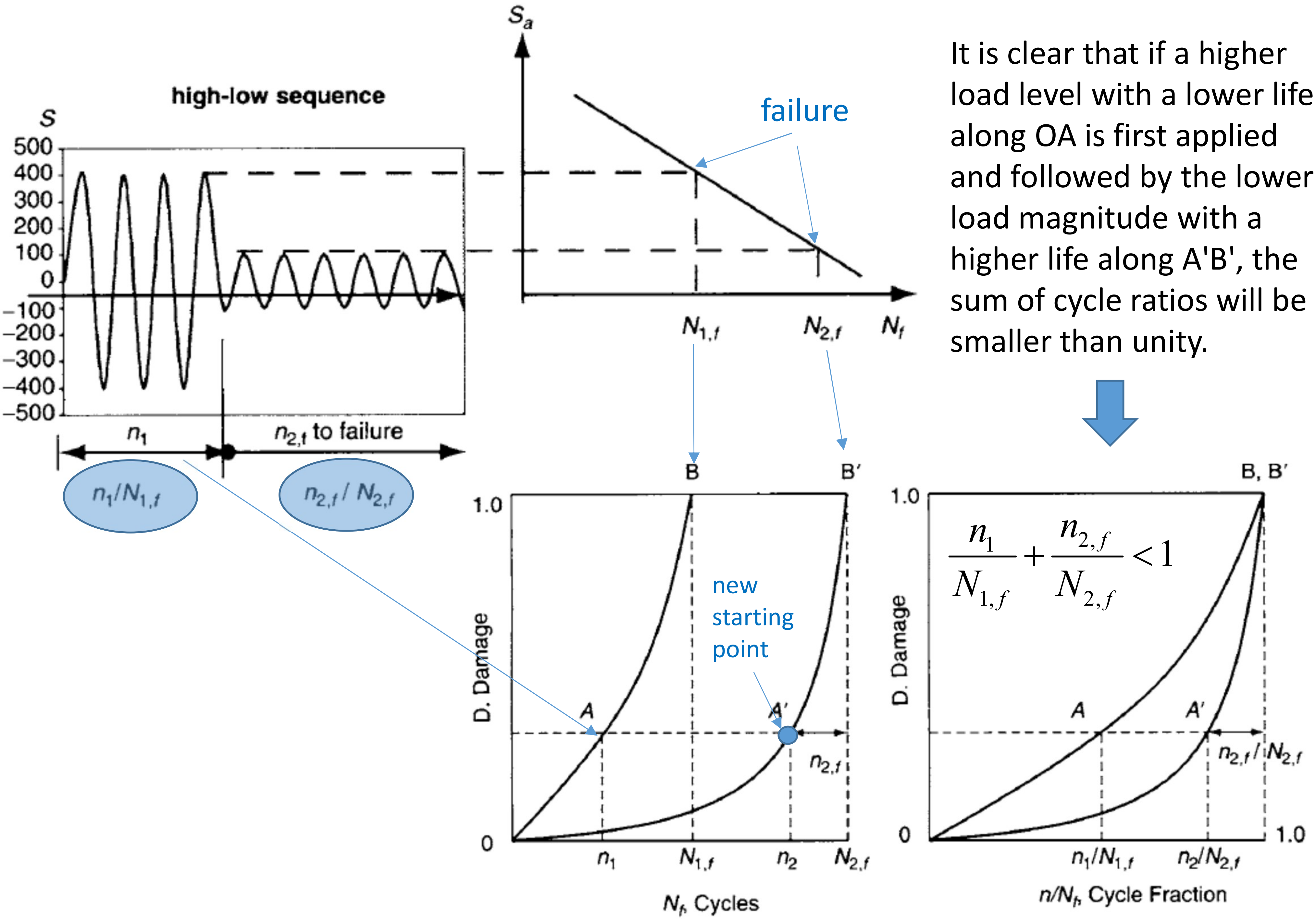
- Increasing crack length -> increasing damage

$$a(n) = a_i + \left(a_f - a_i\right)\left(\frac{n}{N_f}\right)^{\alpha_f}$$



Damage accumulation concept

It is clear that if a higher load level with a lower life along OA is first applied and followed by the lower load magnitude with a higher life along A'B', the sum of cycle ratios will be smaller than unity.



$$\frac{n_1}{N_{1,f}} + \frac{n_{2,f}}{N_{2,f}} < 1$$

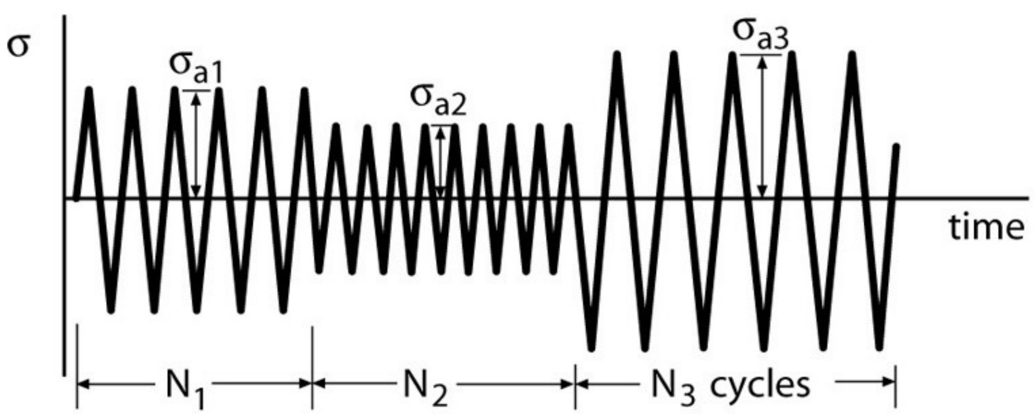
Palmgren-Miner Rule

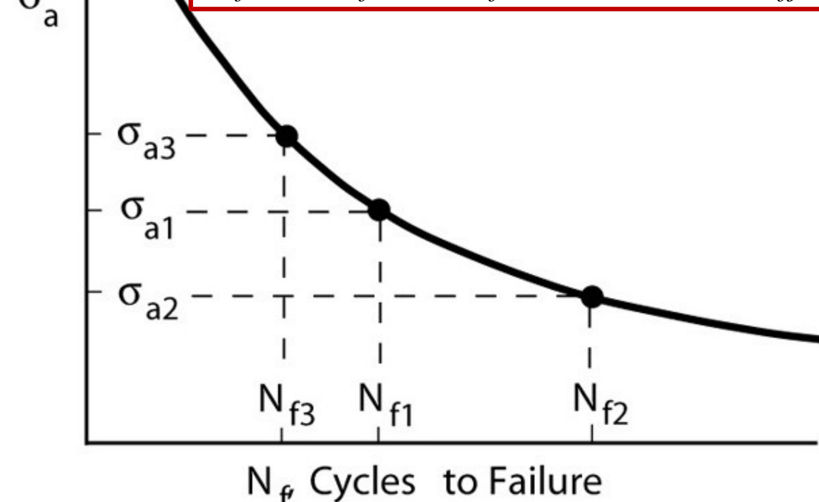
S-N curves from constant amplitude testing can be used to estimate fatigue lives for irregular load-time histories

In 1945, M A Miner advanced a rule that had first been proposed by A. Palmgren in 1924.

The rule called *Miner's rule* or the *Palmgren-Miner linear damage hypothesis*, states that where there are k different stress magnitudes in a spectrum, σ_{ai} $(1 \le i \le k)$, each contributing $N_i(\sigma_{ai})$ cycles, then if $N_{fi}(\sigma_{ai})$ is the number of cycles to failure of a constant stress reversal σ_{ai} (determined by uni-axial fatigue tests), failure occurs when

$$\boxed{\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \ldots\ldots = \sum \frac{N_j}{N_{fj}} = 1}$$

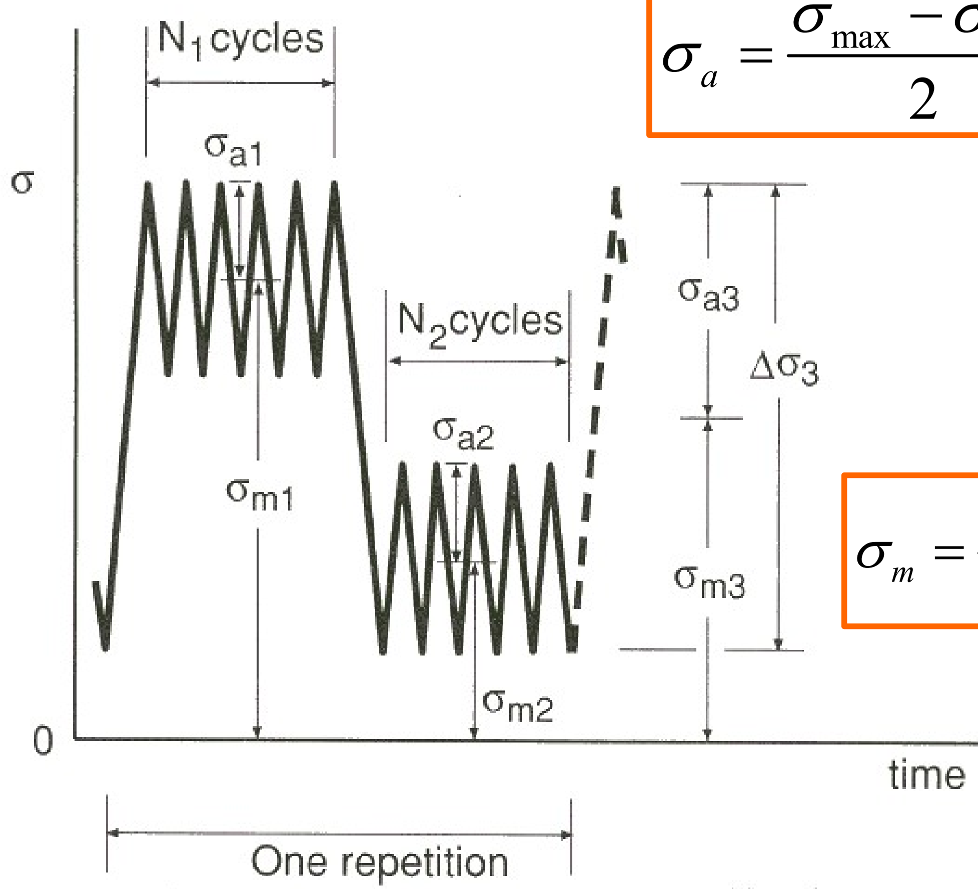




$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{N_3}{N_{f3}} + \ldots = 1$$

Sum life fractions

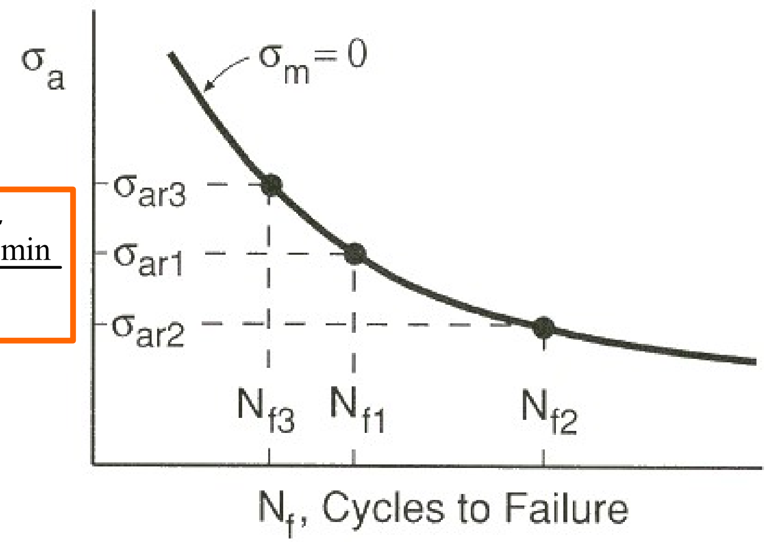
Mean stress and cycle counting



$$\boxed{\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \sigma_f' \left(2N_f\right)^b}$$

$$\boxed{\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}}$$

$$B_f \left[\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} + \frac{1}{N_{f3}} \right] = 1$$



For each *i:*

Stress amplitude

$$\sigma_{ar} = \frac{\sigma_a}{1 - \dfrac{\sigma_m}{\sigma'_f}}$$

mean

Correction for mean

Morrow

Equivalent completely reversed amplitude

$$\sigma_{ar} = \sqrt{\sigma_{\max}\sigma_a}$$

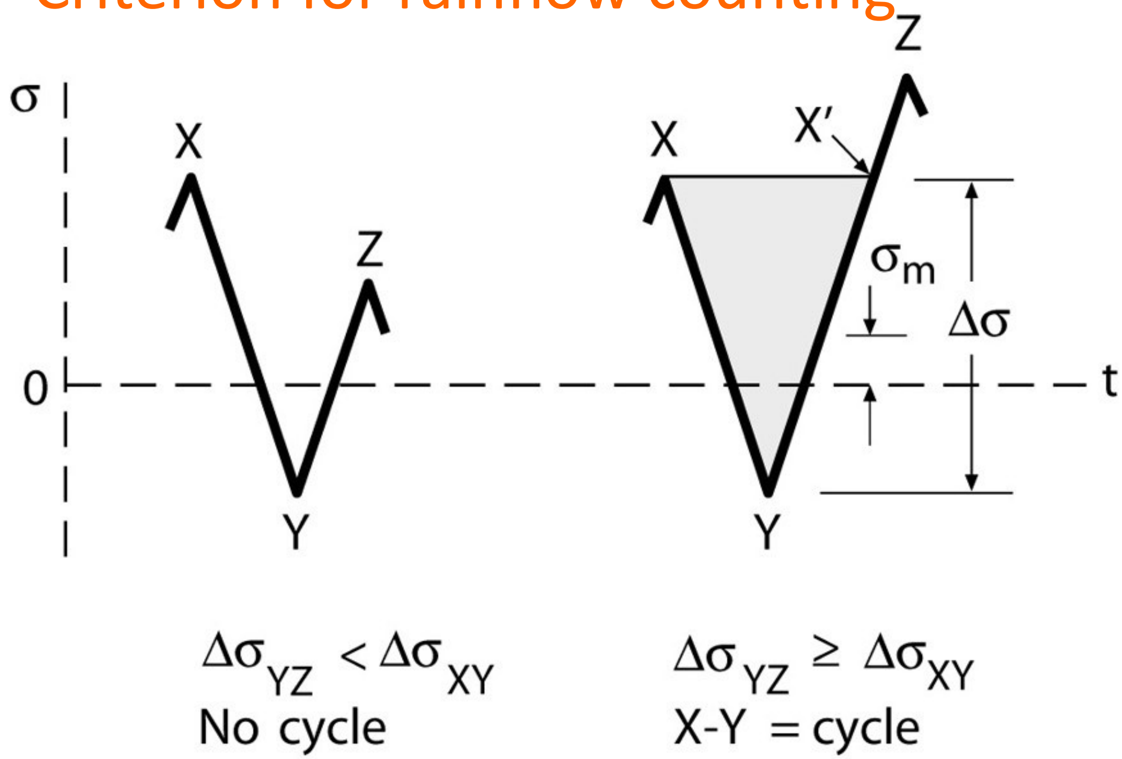
SWT

Rainflow Cycle Counting

The rainflow cycle counting method identifies small events as interruptions of larger events, while also capturing the large event.

The largest cycle counted will be between the highest peak and the lowest valley. The number of cycles is half the number of peak/valley events in the history, not counting the return to the starting point.

Criterion for rainflow counting



<u>For cycle X-Y</u>

Peak: σ_X

Valley: σ_Y

Range: $\Delta\sigma = \sigma_X - \sigma_Y$

Mean: $\sigma_m = (\sigma_X + \sigma_Y)/2$

Comments on Palmgren-Miner Rule

- The Palmgren-Miner rule implicitly assumes that fatigue damage is uniquely related to the life fraction. It does **not** require that any physical measure of fatigue damage must increase linearly.

- A major limitation of the Palmgren-Miner rule is that it does not consider sequence effects, i.e. the order of the loading makes no difference in this rule. Sequence effects are definitely observed in many cases.

- A second limitation is that the Palmgren-Miner rule says that the damage accumulation is independent of stress level.

Factors affecting fatigue

- Testing conditions
  - Mean stress and its significance in fatigue design
  - Frequency
  - Temperature
- Processing conditions
  - Surface finish
  - Residual stresses
- Desine factors
  - Notches
  - Joints
- Metallurgical factors
  - Purity
  - Uniformity
  - Texture
- Correction techniques

Fatigue limit

- The fatigue limit has historically been a prime consideration for long-life fatigue design.
- For a given material the fatigue limit has an enormous range of factors depending on:
  - surface finish,
  - size,
  - type of loading (stress, strain, waveform, frequency),
  - mean stresses,
  - temperature,
  - corrosive, and other aggressive environments,
  - residual stresses
  - stress concentrations

Preparation

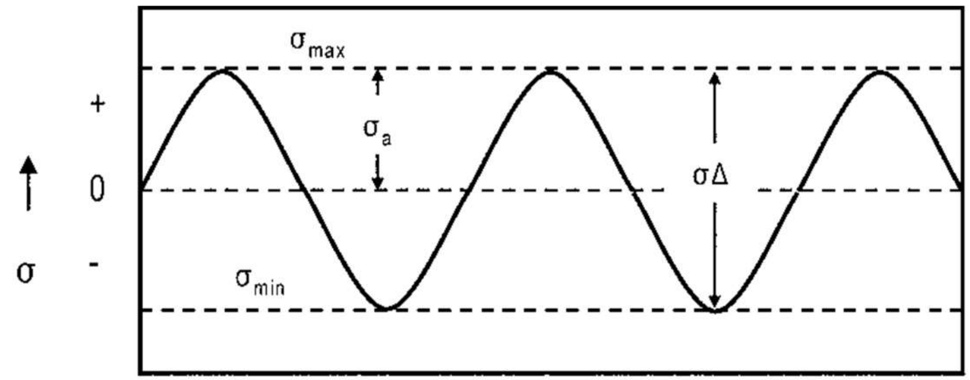
Operational

Metallurgical

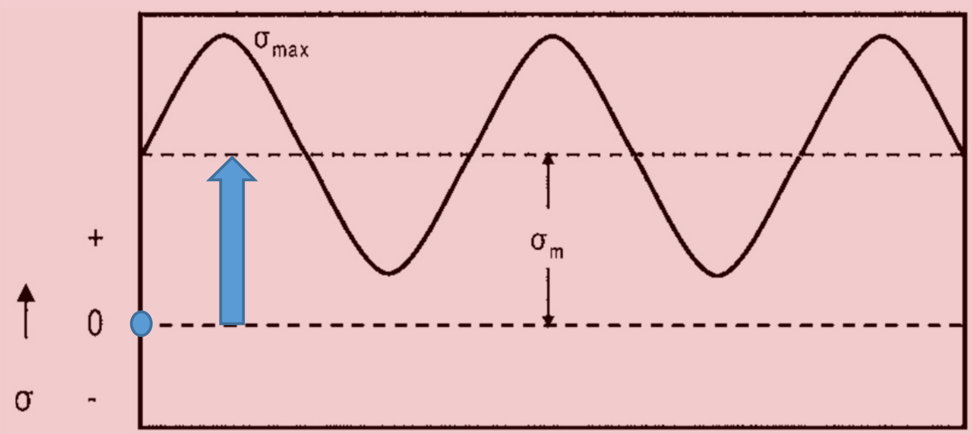
Mean Stress effect

- SN curves are most often presented for a fully reversed test. This means that the stresses applied cycle between equal tensile and compressive states. In realistic structural loading, it is more common for the cyclic loads to oscillate around a non-zero mean state. This non-zero mean state has a significant effect on the life to failure.

- A method is required to account for the presence of a mean stress in the cycle when using the standard SN data for fatigue design



Fully Reversed Loading

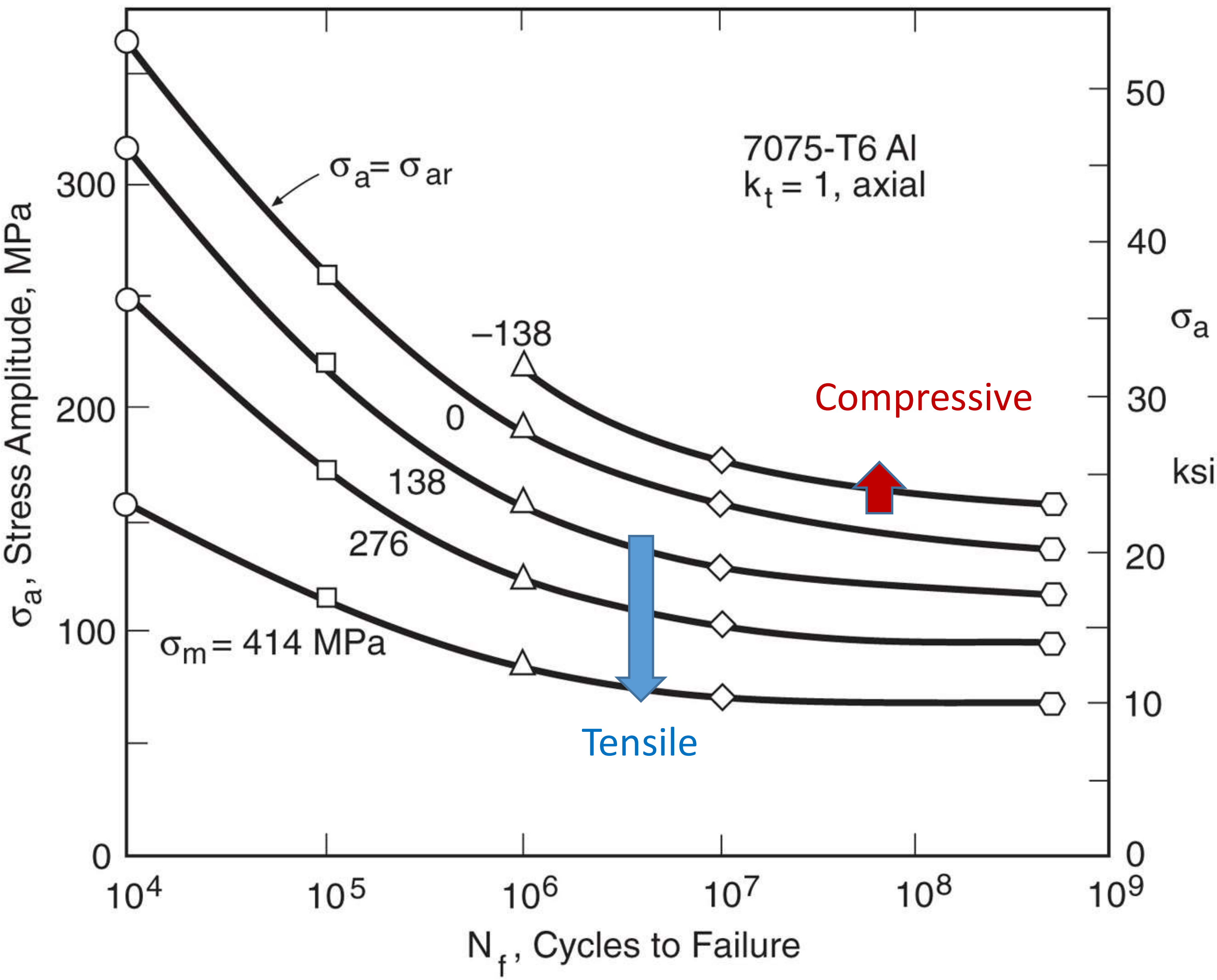


Tension -Tension with Applied Stress

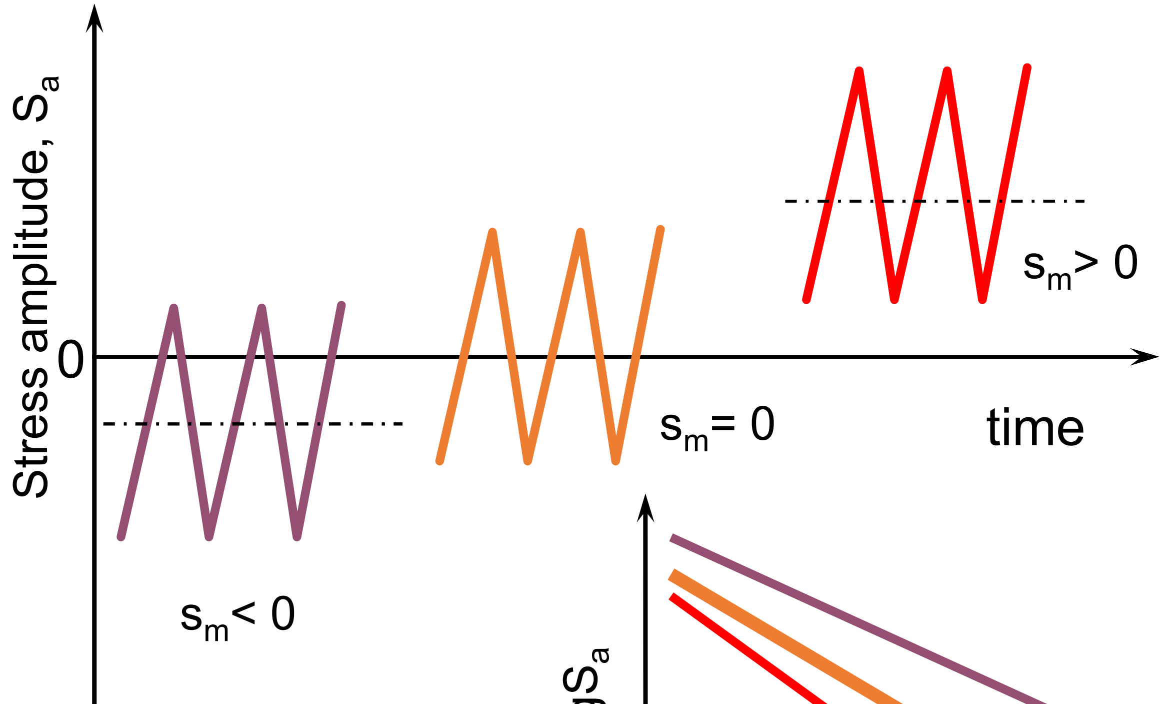
Effect of R-ratio (mean stress)

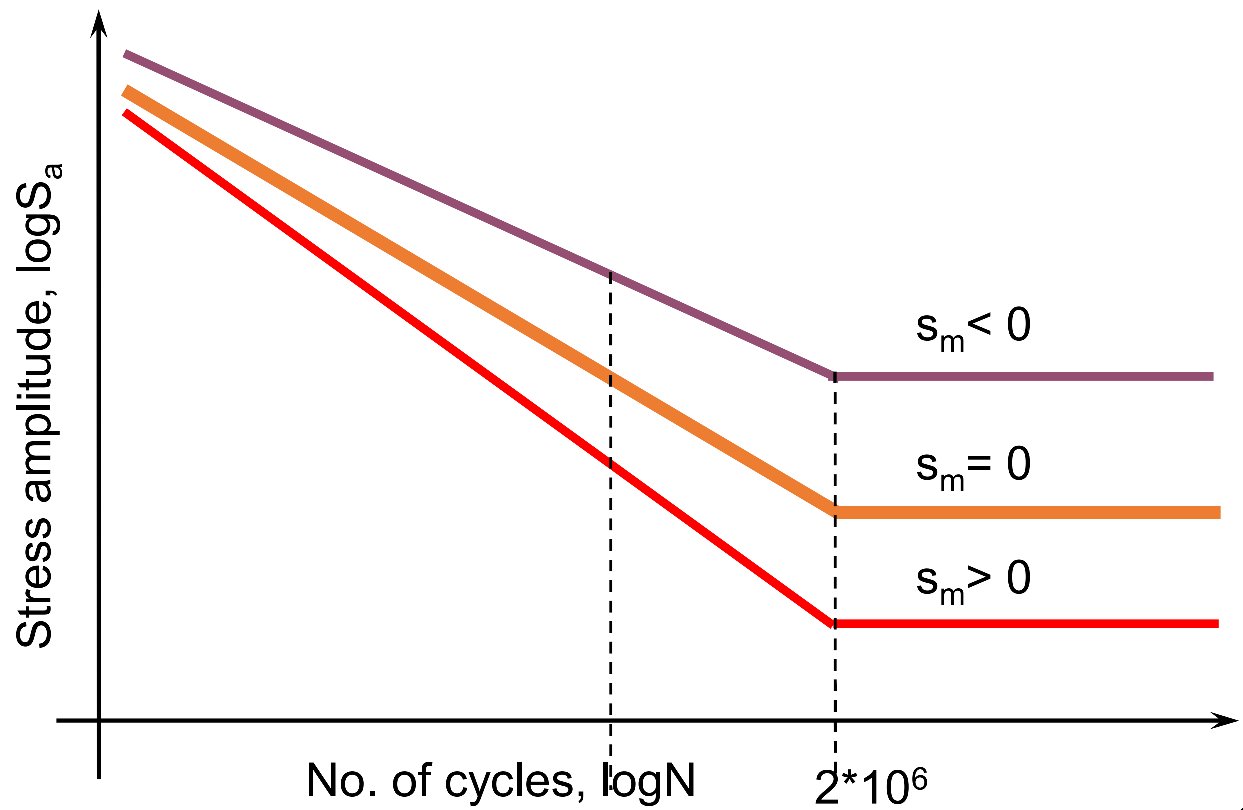
- Fatigue life depends heavily on R ratio (ratio of maximum to minimum stress)
- High R ratio means maximum stresses are higher for same amplitude, hence faster crack growth
- Can use Gerber, Goodman, Soderberg or constant life diagrams to account for these effects
- Diagrams show life at various stress amplitudes and R- ratios

MEAN STRESS EFFECTS



Mean Stress Effect





Fatigue Limit – Modifying Factors

For many years the emphasis of most fatigue testing was to gain an empirical understanding of the effects of various factors on the base-line S-N curves for ferrous alloys in the intermediate to long life ranges. The variables investigated include:

- **Rotational bending fatigue limit, S_e',**
- **Surface conditions, k_s,**
- **Size, k_b,**
- **Mode of loading, k_c,**
- **Temperature, k_d**
- **Reliability factor, k_e**
- **Miscellaneous effects (notch), k_f**

Fatigue limit of a machine part, S_e

$$S_e = k_s\, k_b\, k_c\, k_d\, k_e\, k_f \cdot S_e'$$

Size Effects on Endurance Limit

Fatigue is controlled by the weakest link of the material, with the probability of existence (or density) of a weak link increasing with material volume. The size effect has been correlated with the thin layer of surface material subjected to 95% or more of the maximum surface stress.

There are many empirical fits to the size effect data. A fairly conservative one is:

$$k_b = \frac{S_e}{S_e^{'}} \quad \begin{cases} 1.0 & d \le 8mm \\ 1.189^{-0.097} & 8 \le d \le 250mm \end{cases}$$

- The size effect is seen mainly at very long lives.
- The effect is small in diameters up to 50 mm (even in bending and torsion).

Size Effects on Endurance Limit

- In the case of non-circular members the approach is based on so called effective diameter, d_e. The effective diameter, d_e, for non-circular cross sections is obtained by equating the volume of material stressed at and above 95% of the maximum stress to the same volume in the rotating-bending specimen.

$$A_{0,95\sigma} = \pi\left(\frac{d^2}{4} - \frac{(0.95d)^2}{4}\right) = 0.0766d^2$$

$$d_e = \sqrt{\frac{A_{0,95\sigma}}{0.0766}}$$

Loading Effects on Endurance Limit

The ratio of endurance limits for a material found using axial and rotating bending tests ranges from 0.6 to 0.9.

$$S_{e(axial)} \approx (0.7-0.9)S_{e(bending)}$$

$$k_c = 0.7-0.9 \quad (suggested\ by\ Shigley\ k_c = 0.85)$$

The ratio of endurance limits found using torsion and rotating bending tests ranges from 0.5 to 0.6. A theoretical value obtained from von Mises-Huber-Hencky failure criterion is been used as the most popular estimate.

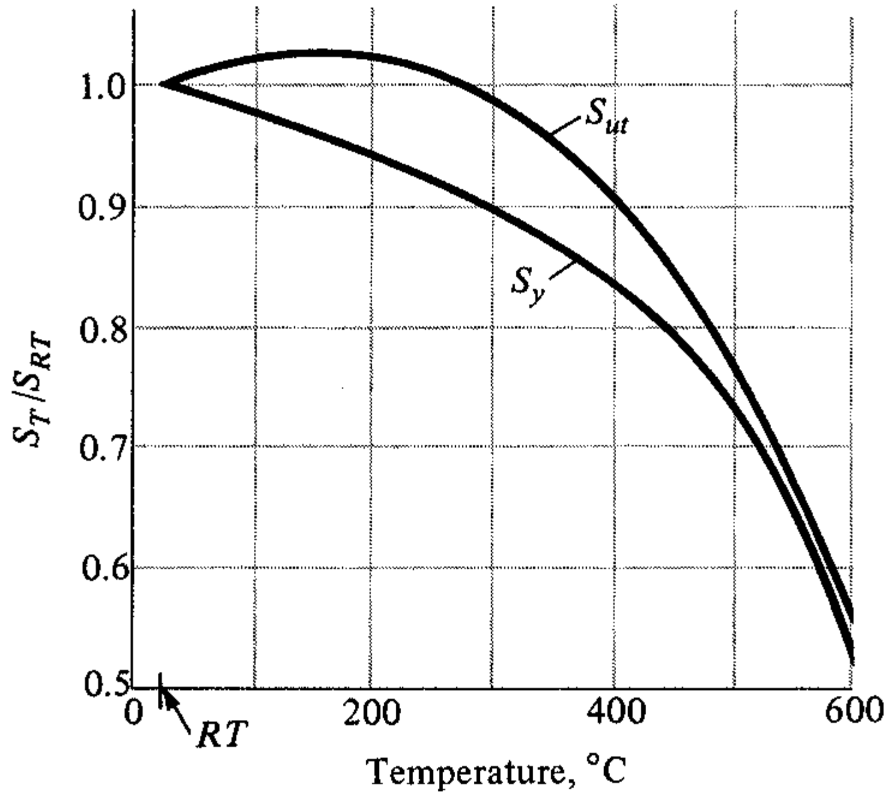
$$\tau_{e(torsion)} \approx 0.577 S_{e(bending)}$$

$$k_c = 0.57 (suggested\ by\ Shigley\ k_c = 0.59)$$

R. Budinas, J.K. Nisbett, Shigley's Mechanical Engineering Design, Mcgraw-Hill series, 2015

Temperature Effect

A plot of the results of 145 tests of 21 carbon and alloy steels showing the effect of operating temperature on the yield strength S_y and the ultimate strength S_{ut}. The ordinate is the ratio of the strength at the operating temperature to the strength at room temperature. The standard deviations were $0.0442 \le \hat{\sigma} \le 0.152$ for S_y and $0.099 \le \hat{\sigma} \le 0.110$ for S_{ut}. *[Data source: E. A. Brandes (ed.), Smithells Metals Reference Book, 6th ed., Butterworth, London, 1983, pp. 22-128 to 22-131.]*



From: Shigley and Mischke, Mechanical Engineering Design, 2001

$$S_{e,T} = S_{e,RT} k_d = S_{e,RT} \frac{S_{u,T}}{S_{u,RT}}; \qquad k_d = \frac{S_{u,T}}{S_{u,RT}}$$

Reliability factor k_e

The reliability factor accounts for the scatter of reference data such as the rotational bending fatigue limit S_e'.

The estimation of the reliability factor is based on the assumption that the scatter can be approximated by the normal statistical probability density distribution.

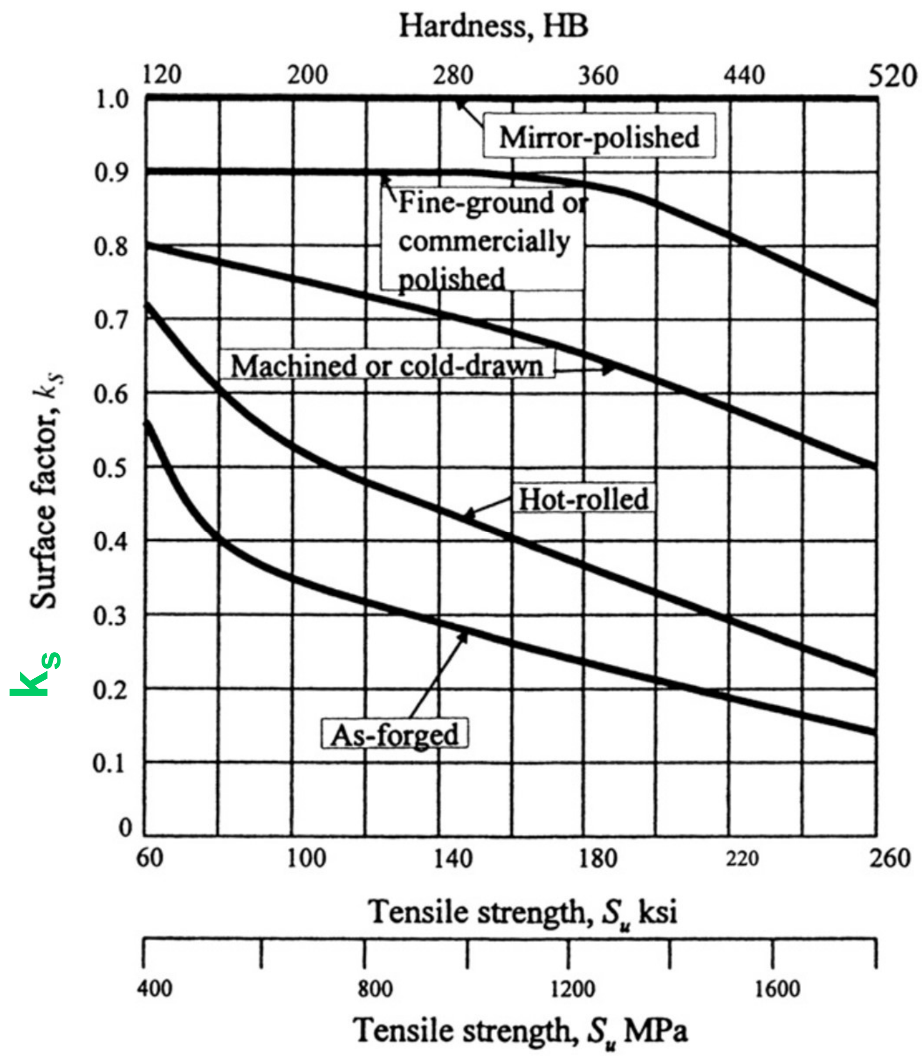
$$k_e = 1 - 0.08 \times z_a$$

The values of parameter z_a associated with various levels of reliability can be found (Shigley et.al.)

R. Budinas, J.K. Nisbett, Shigley's Mechanical Engineering Design, Mcgraw-Hill series, 2015

Surface Finish Effects on Fatigue Limit

The scratches, pits and machining marks on the surface of a material add stress concentrations to the ones already present due to component geometry. The correction factor for surface finish is sometimes presented on graphs that use a qualitative description of surface finish such as “polished” or “machined”.



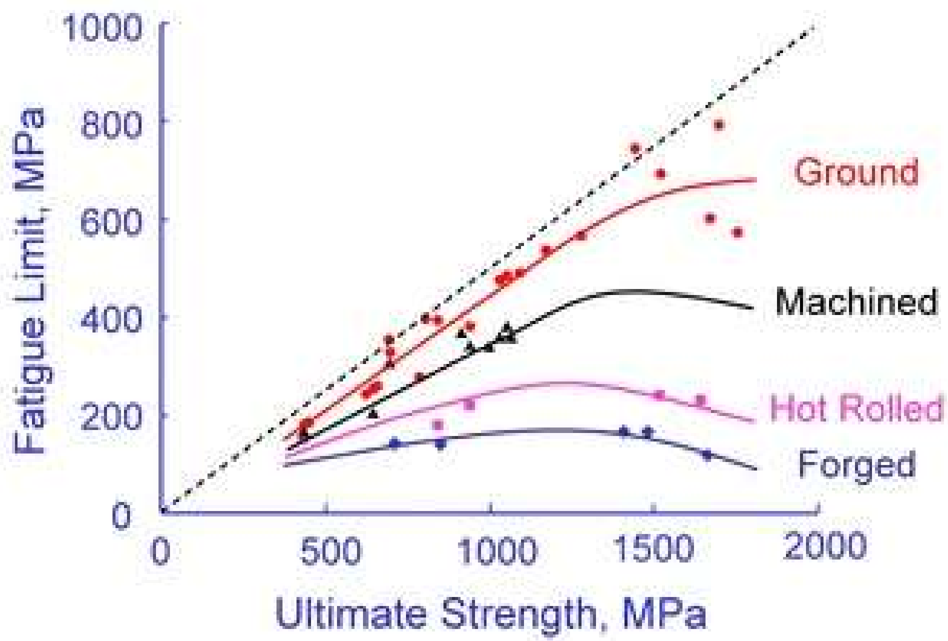
Below a generalized empirical graph is shown which can be used to estimate the effect of surface finish in comparison with mirror-polished specimens [Shigley].

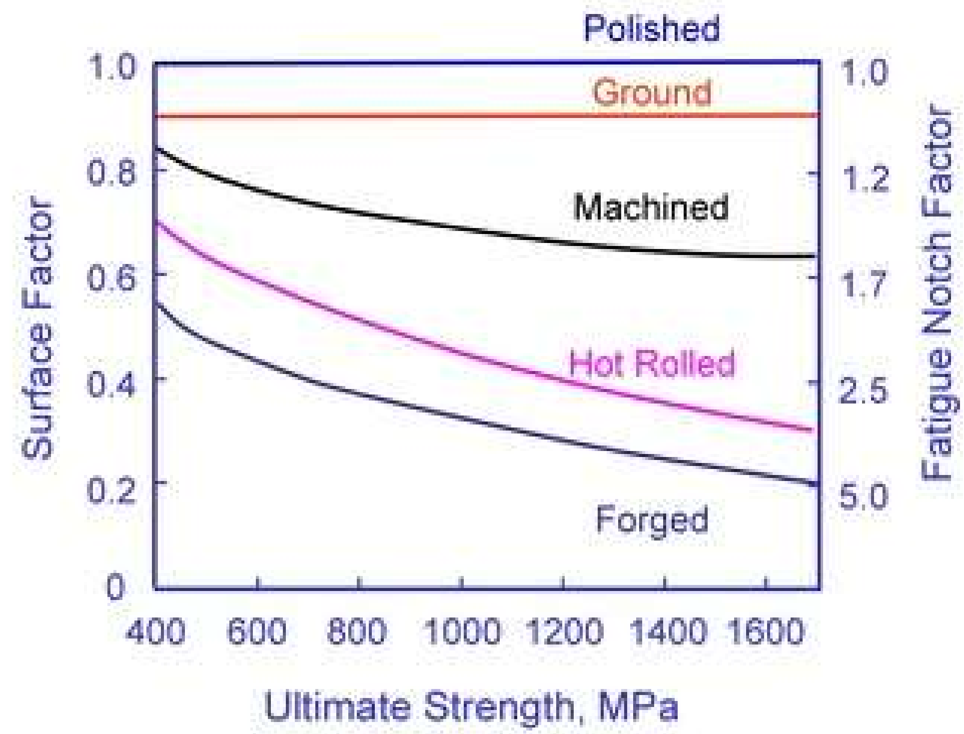
R. Budinas, J.K. Nisbett, Shigley's Mechanical Engineering Design, Mcgraw-Hill series, 2015

Effect of various surface finishes on the fatigue limit of steel. Shown are values of the k_s, the ratio of the fatigue limit to that for polished specimens.

(from R. Stephens, A. Fatemi, Metal Fatigue in Engineering, Wiley &Sons 2012)

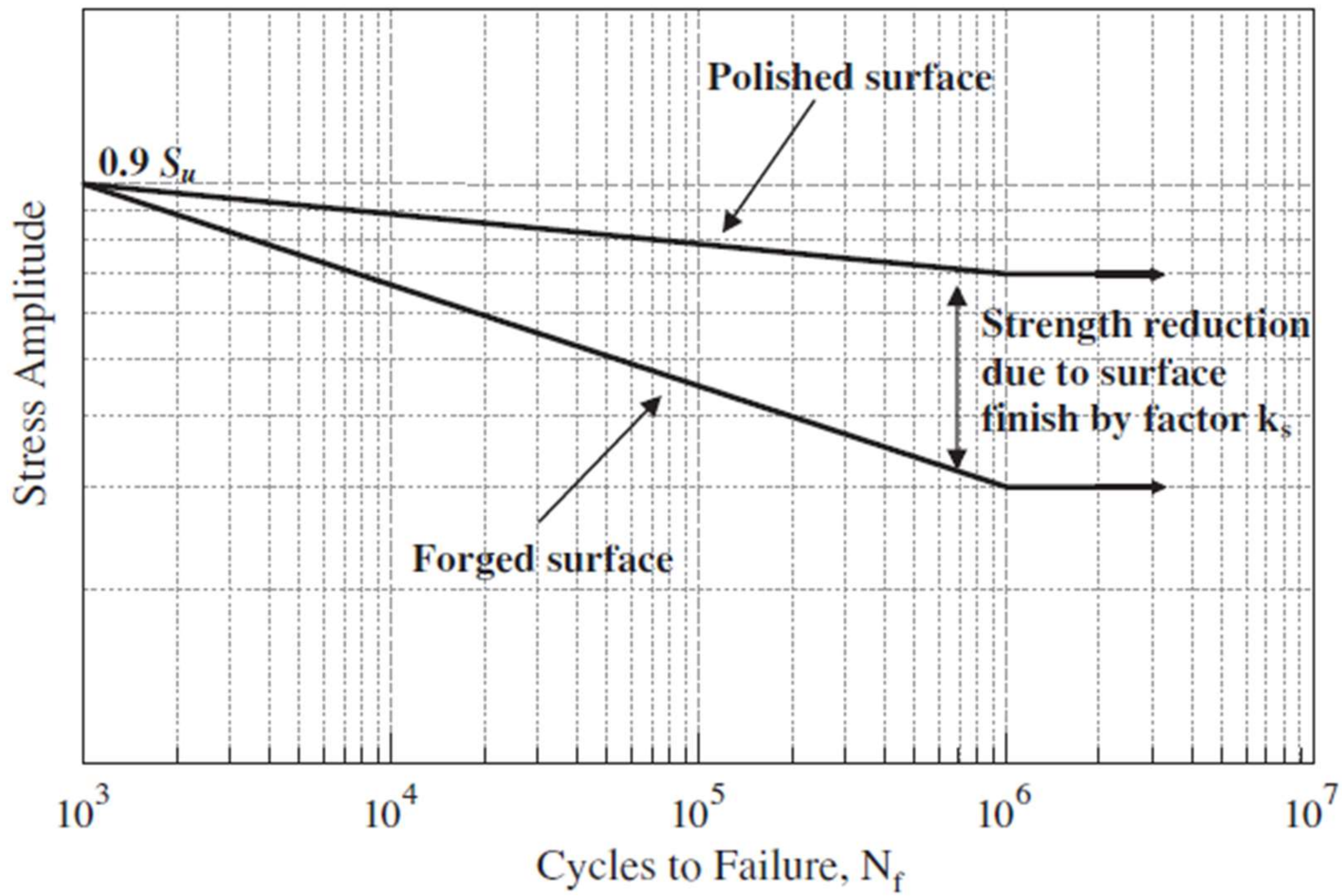
Surface Finish Effects on Fatigue Limit



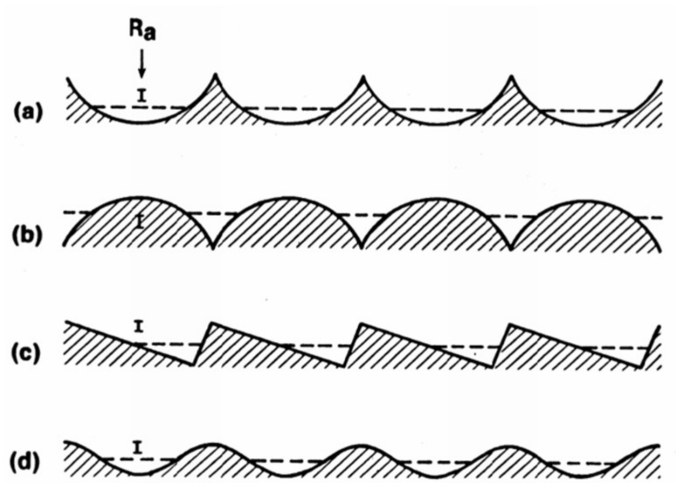


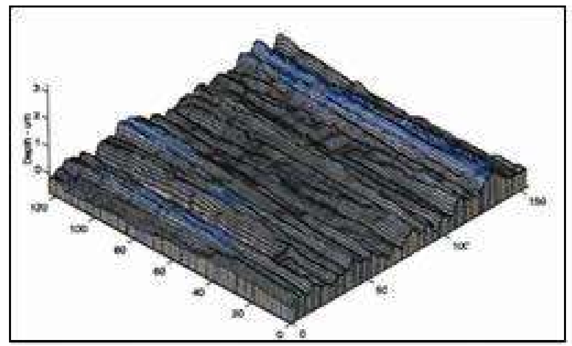
From Noll and Lipson, "Allowable Working Stresses", Society for Experimental Stress Analysis, Vol. III, 1949

Surface Finish Effects on Fatigue Limit

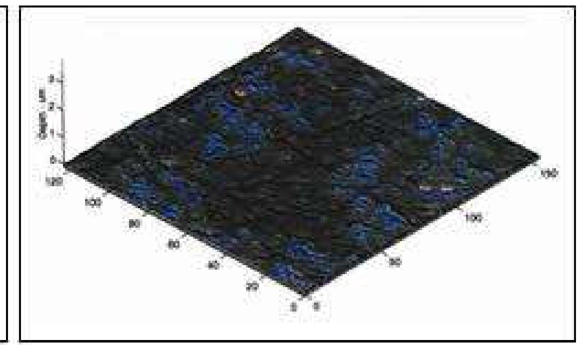


Surface Finish Effects on Fatigue Limit

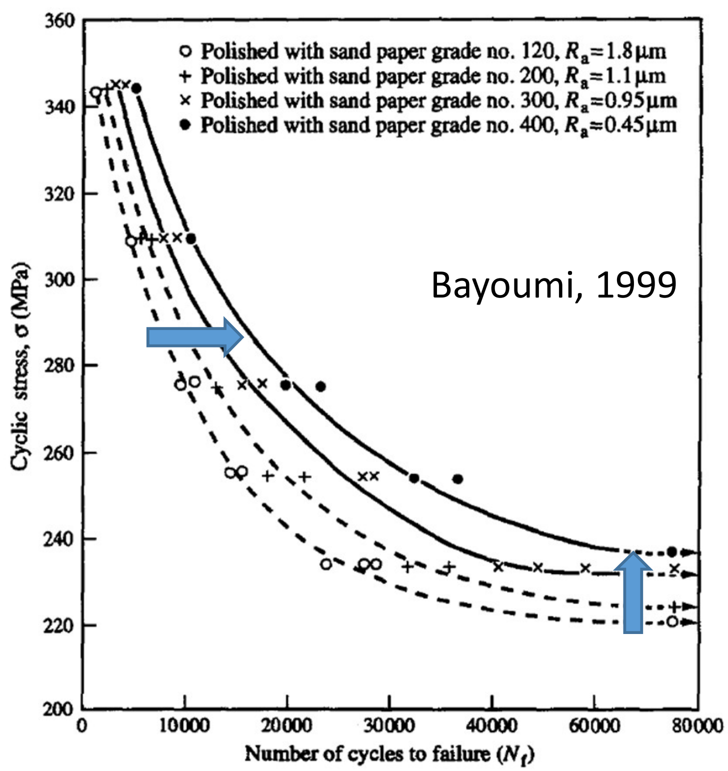




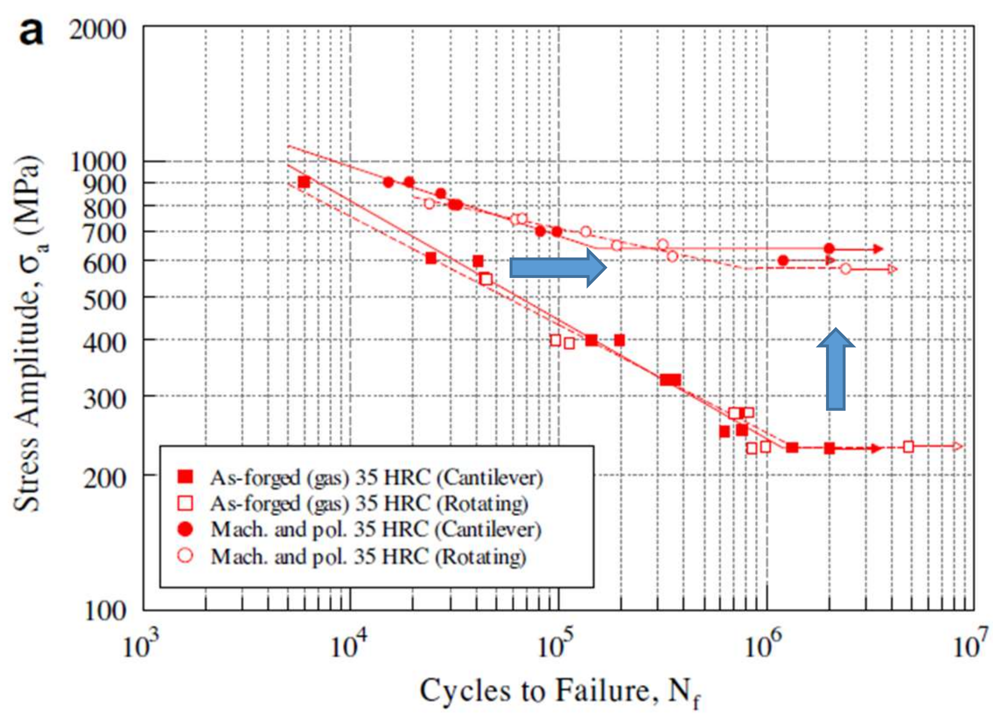
grinded



polished

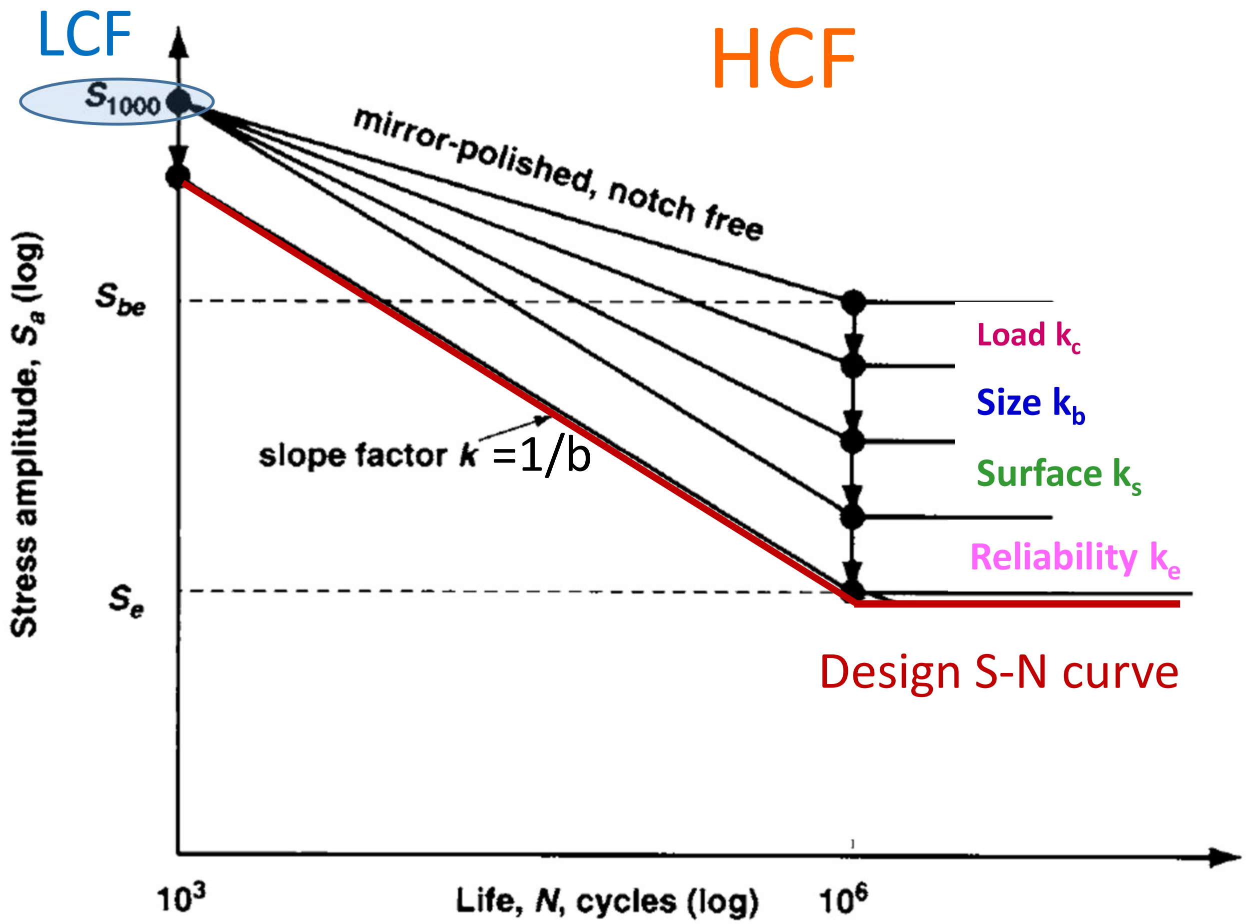


Bayoumi, 1999



Fatemi, 2012

Effect of Different factors on Fatigue: Summary and Design Problem



+ Temperature,
  Environment
  Mean stress
  Notch

$$k = \prod_i k_i = k_B k_C k_S k_e \ldots$$

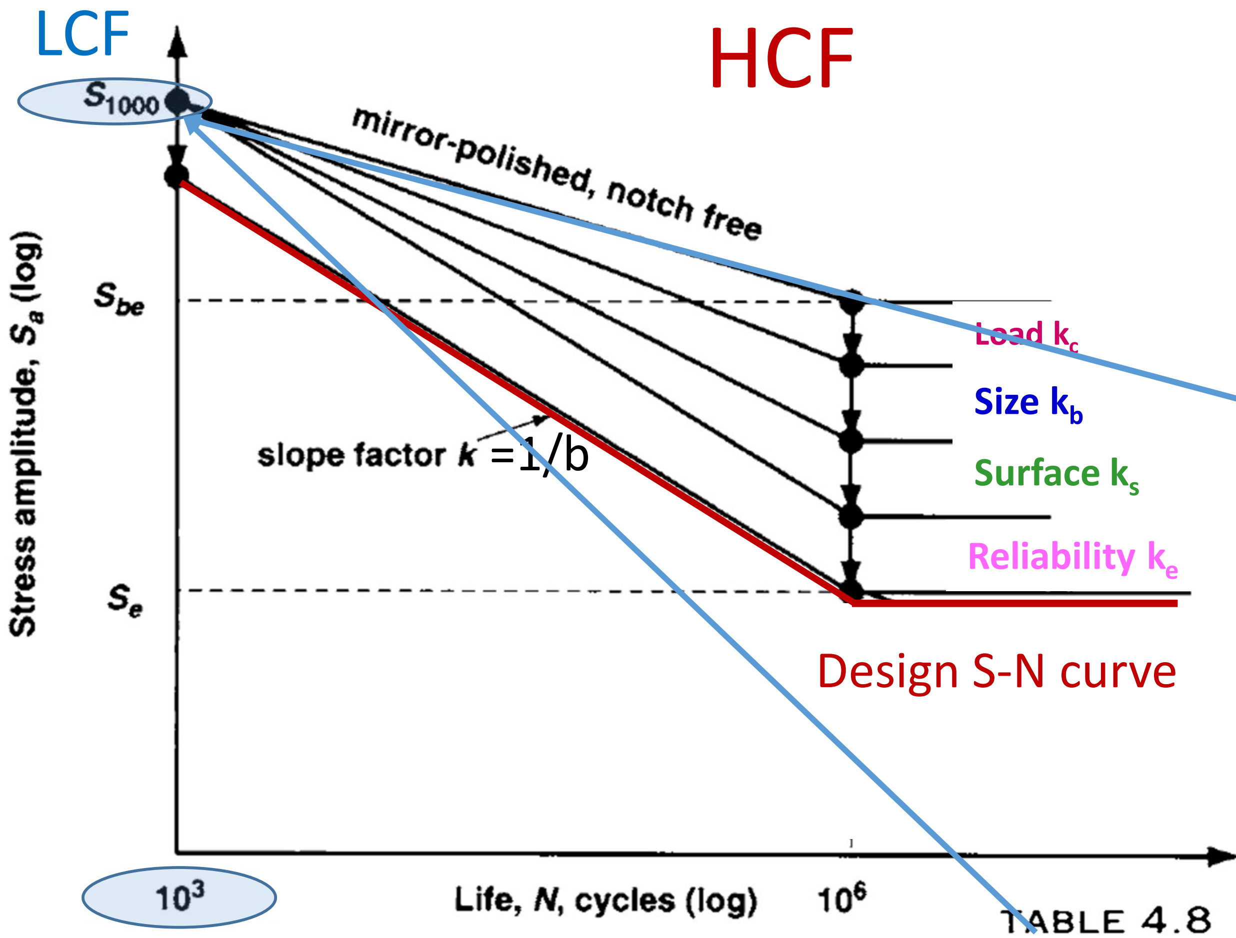
$$S_e \equiv \sigma_{-1} = k_B k_C k_S k_e \ldots S_{be}$$

modifying factors are empirically based and usually range from 0.0 to 1.0.

Stress based fatigue: Design Problem

Generate the *S*-*N* curve with 90% of reliability for a forged steel shaft under torsional loading. The shaft has a diameter of 20 mm and an ultimate strength UTS of 1000 MPa.

Fatigue Design Curve



1) Exploit the relation between tensile and fatigue properties

Se vs UTS
or
S_{1000} vs UTS

2) Then CORRECT it for known factors

TABLE 4.8 Estimates of S_{1000}

Type of Material	Type of Loading	S_{1000}
All	Bending	$0.9 \times S_u$
All	Axial	$0.75 \times S_u$
Steel	Torsion	$0.9 \times S_{us} = 0.72 \times S_u$
Nonferrous	Torsion	$0.9 \times S_{us} = 0.63 \times S_u$
Cast iron	Torsion	$0.9 \times S_{us} = 1.17 \times S_u$

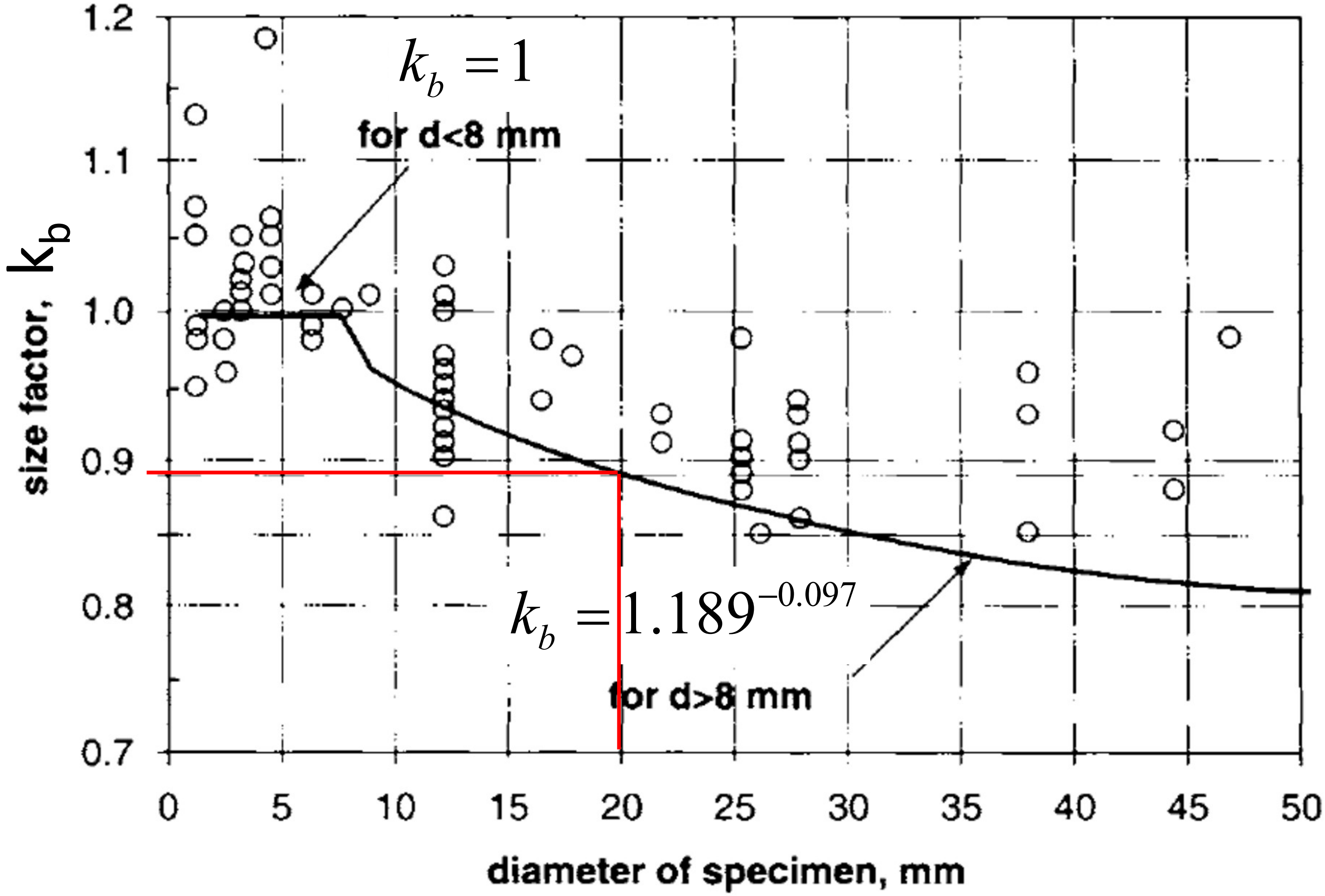
Fatigue – UTS relation

TABLE 4.10 Estimates of Baseline Bending Fatigue Limits for Various Materials (From Juvinall, 1967)

	S_{be}	@ Cycles	Comments
Type of Material			
Microstructure of Steels			
Steel - Ferrite	$0.58 \times S_u$	10^6	
Steel – Ferrite + Pearlite	$0.38 \times S_u$	10^6	
Steel - Pearlite	$0.38 \times S_u$	10^6	
Steel – Untempered martensite	$0.26 \times S_u$	10^6	
Steel – Highly tempered Martensite	$0.55 \times S_u$	10^6	
Steel – Highly Tempered Martensite + Tempered Bainite	$0.5 \times S_u$	10^6	
Steel – Tempered Bainite	$0.5 \times S_u$	10^6	
Steel - Austenite	$0.37 \times S_u$	10^6	
Type of Material			
Wrought Steels	$0.5 \times S_u$	10^6	$S_u < 1400$ MPa
Wrought Steels	700 MPa	10^6	$S_u \geq 1400$ MPa
Cast iron	$0.4 \times S_u$	5×10^7	-
Aluminum alloys	$0.4 \times S_u$	5×10^8	$S_u < 336$ MPa
Aluminum alloys	130 MPa	5×10^8	$S_u \geq 336$ MPa
PM cast aluminum	80 MPa	5×10^8	-
Sand cast aluminum	55 MPa	5×10^8	-

ref: Y-L. Lee, et al. Fatigue Testing and Analysis Theory and Practice, Elsevier (Netherlands), 2005

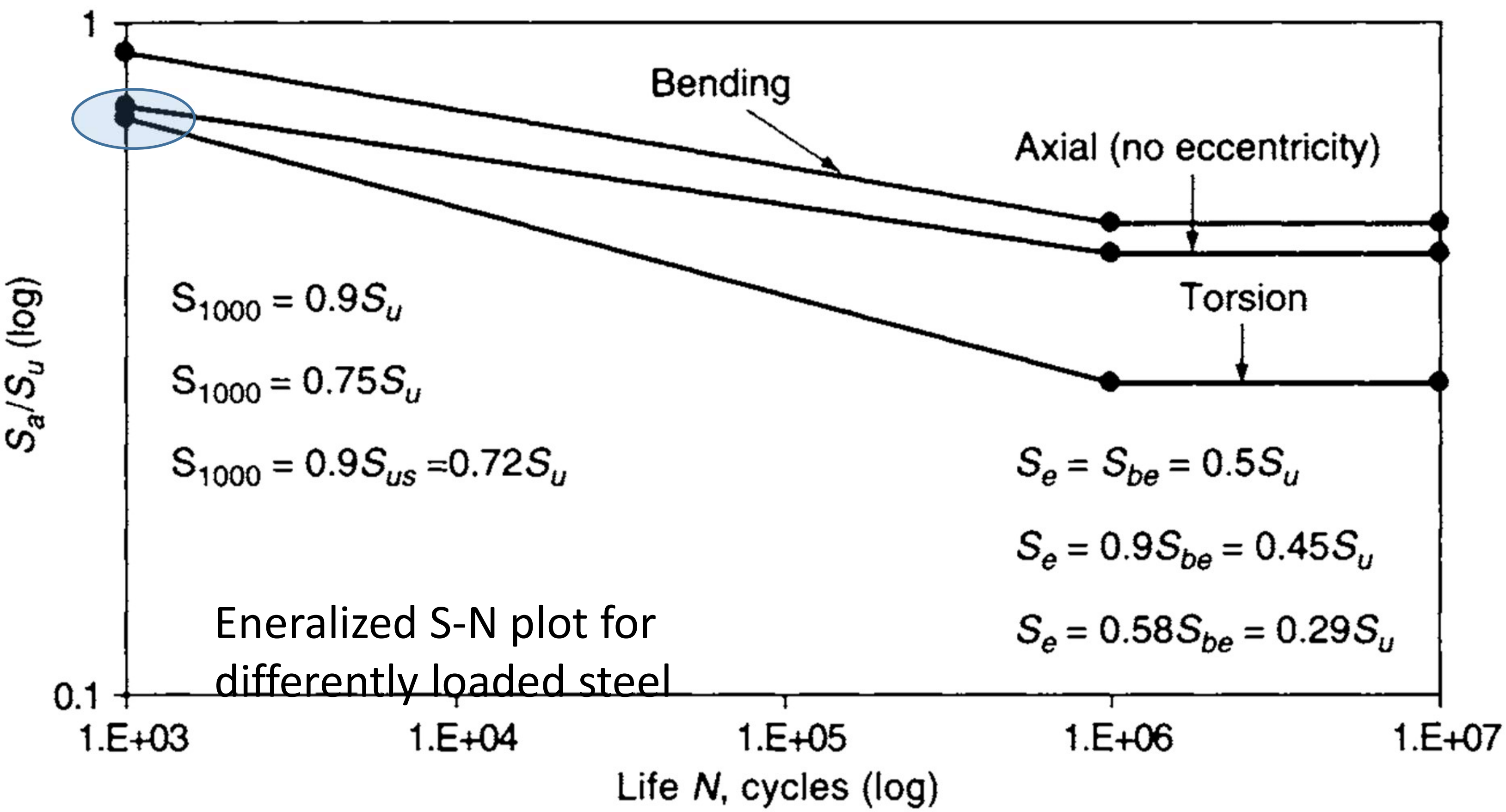
Size Effect



$$k_b = \frac{S_e}{S_e^{'}} \quad \begin{cases} 1.0 & d \le 8mm \\ 1.189^{-0.097} & 8 \le d \le 250mm \end{cases}$$

ref: Y-L. Lee, et al. Fatigue Testing and Analysis Theory and Practice, Elsevier (Netherlands), 2005

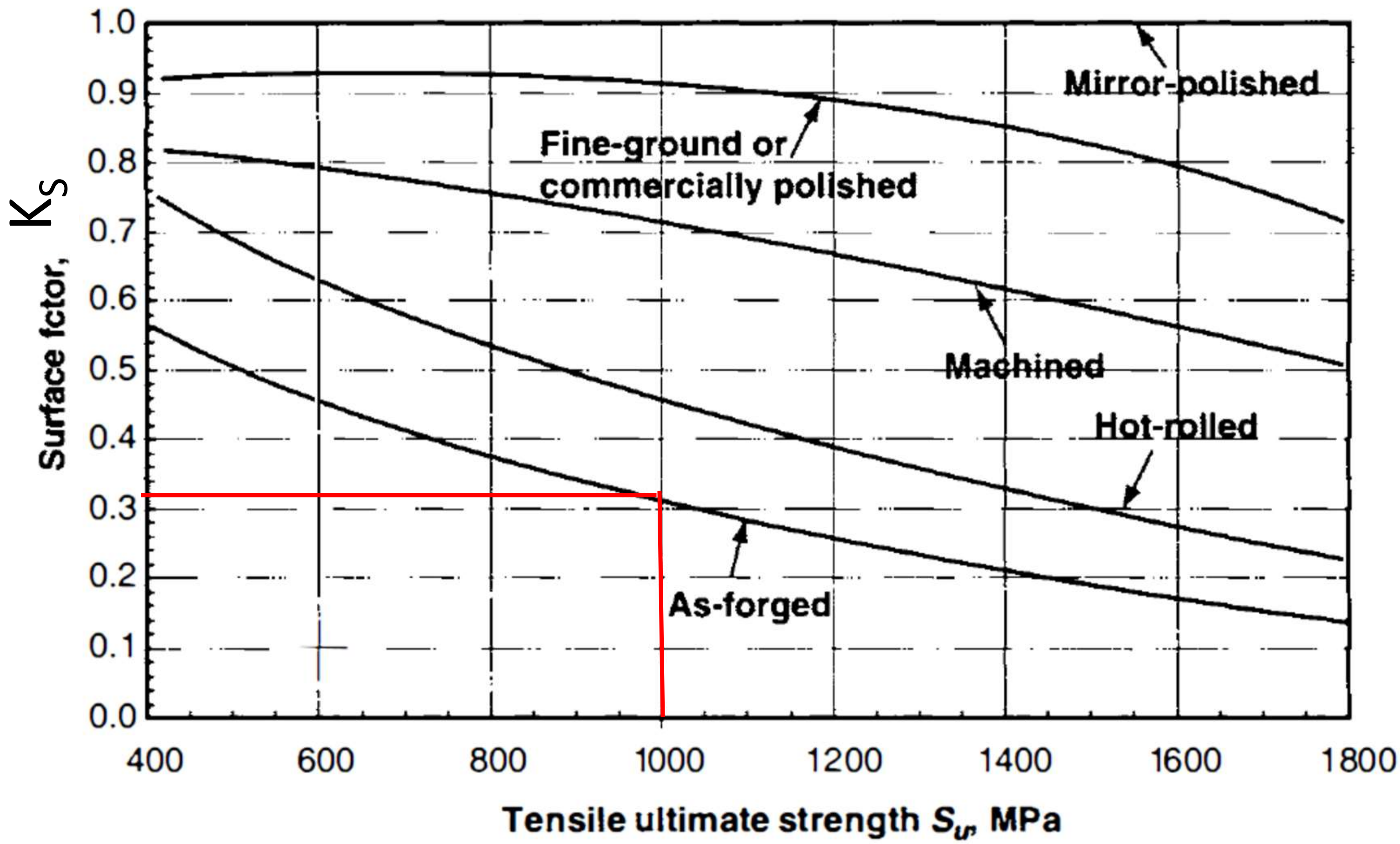
Loading Mode Effect



Eneralized S-N plot for differently loaded steel

Type of Loading	K_C	Comments
Pure axial loading	0.9	
Axial loading (with slight bending)	0.7	
Bending	1.0	
Torsional	0.58	For steels
Torsional	0.8	For cast iron

Surface Effect



ref: Y-L. Lee, et al. Fatigue Testing and Analysis Theory and Practice, Elsevier (Netherlands), 2005

Reliability

Computed based on the assumption of Normality of fatigue strength distribution in the HCF regime

Reliability	Ke
0.50	1.000
0.90	0.897
0.95	0.868
0.99	0.814
0.999	0.753
0.9999	0.702
0.99999	0.659
0.999999	0.620

See optional “Useful readings” for derivation

ref: Y-L. Lee, et al. Fatigue Testing and Analysis Theory and Practice, Elsevier (Netherlands), 2005

Solution:

The fatigue strength at 10^3 cycles (S_{1000}) depends on the reliability level and the type of loading. For example

$$S_{1000,\ RB} = S_{1000} \times K_e \quad (Kc=1)$$

For torsional loading, the fatigue strength S_{1000} is estimated as

$$S_{1000,\ T} = 0.9x0.8\ \sigma_{UTS}\ K_e = 646\ \text{MPa}$$

Ke=0.897

For torsional stress the fatigue limit is estimated as

$$S_e = KcKbKsKeS_{be}$$

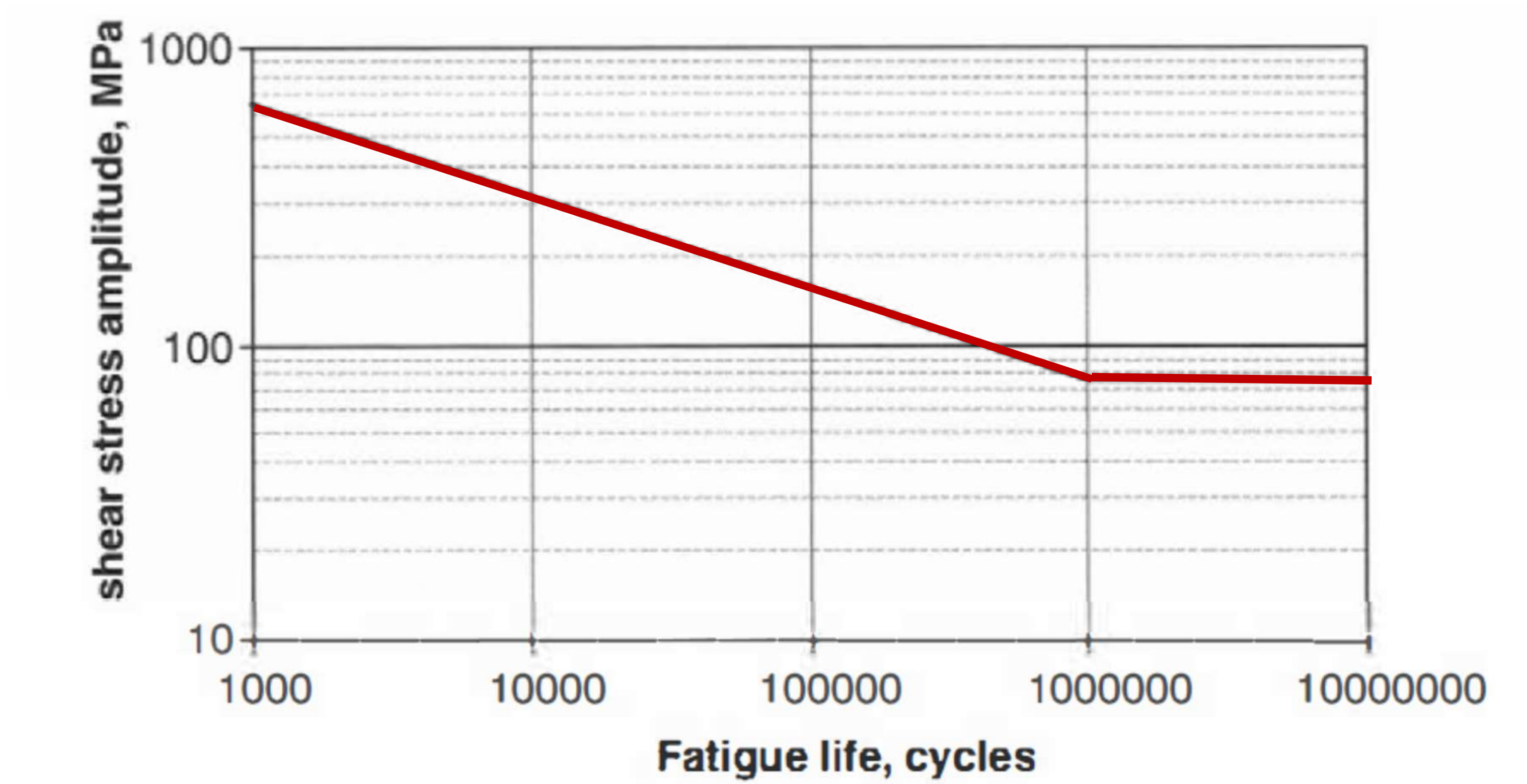
with the bending fatigue limit S_{be} for wrought steels with UTS = 1000 MPa is S_{be}= 0.5 σ_{UTS} = 500 MPa

Load factor for ductile steels in torsion Kc=0.58
Size factor K_b=0.89
Surface Finish factor for forged steel having 1000 MPa UTS Ks=0.33

Therefore, the fatigue limit of the shaft under torsional loading is

$$S_e = (500\text{ MPa})(0.58)(0.89)(0.33)(0.897) = 76.4\text{ MPa}$$

After both S1000 and Se have been determined, they can be plotted to estimate the design S-N curve



Also, the fatigue strength at a specific fatigue life can be determined.

Resume

“You can teach a student a lesson for a day; but if you can teach him to learn by creating curiosity, he will continue the learning process as long as he lives”.

Clay P. Bedford