Fatigue, Damage and Failure of Composite Materials: Mechanisms, Fatigue Life Diagrams and Life Prediction

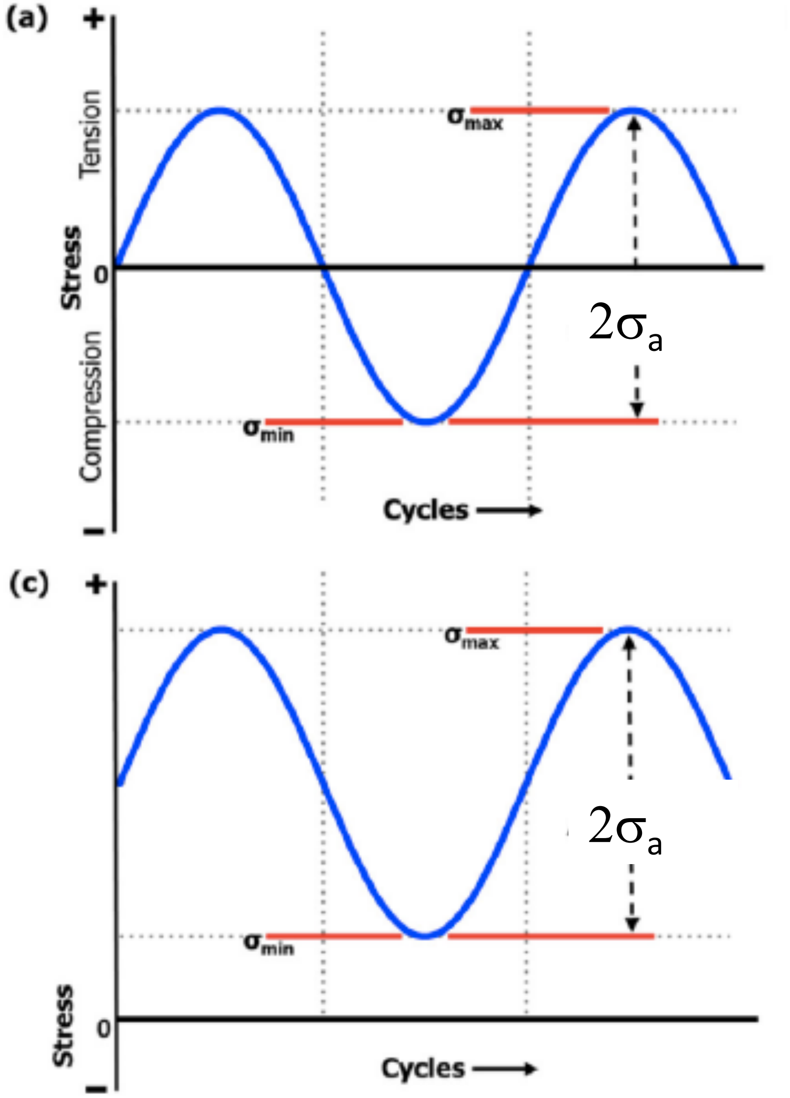
Ramesh Talreja
Department of Aerospace Engineering
Department of Materials Science and Engineering
Texas A&M University, College Station, Texas, USA

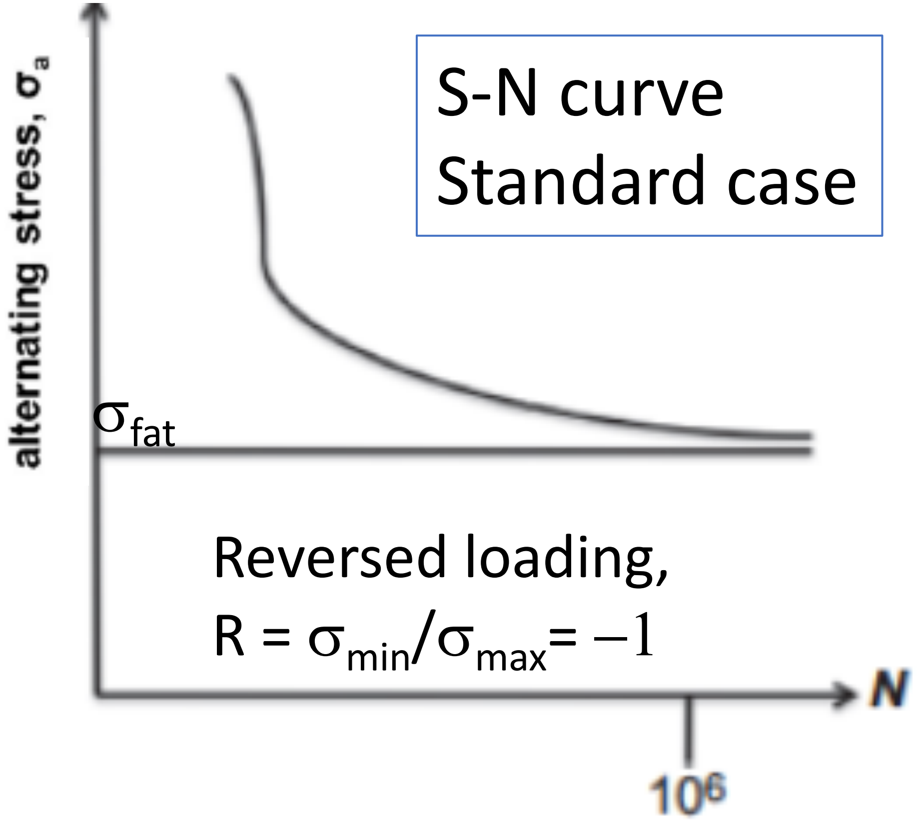
UTMIS Autumn Course, Gothenburg, Sweden, 15-16 October 2019

Lecture 4 : FATIGUE LIFE PREDICTION

Contents

Empirical models – metal fatigue

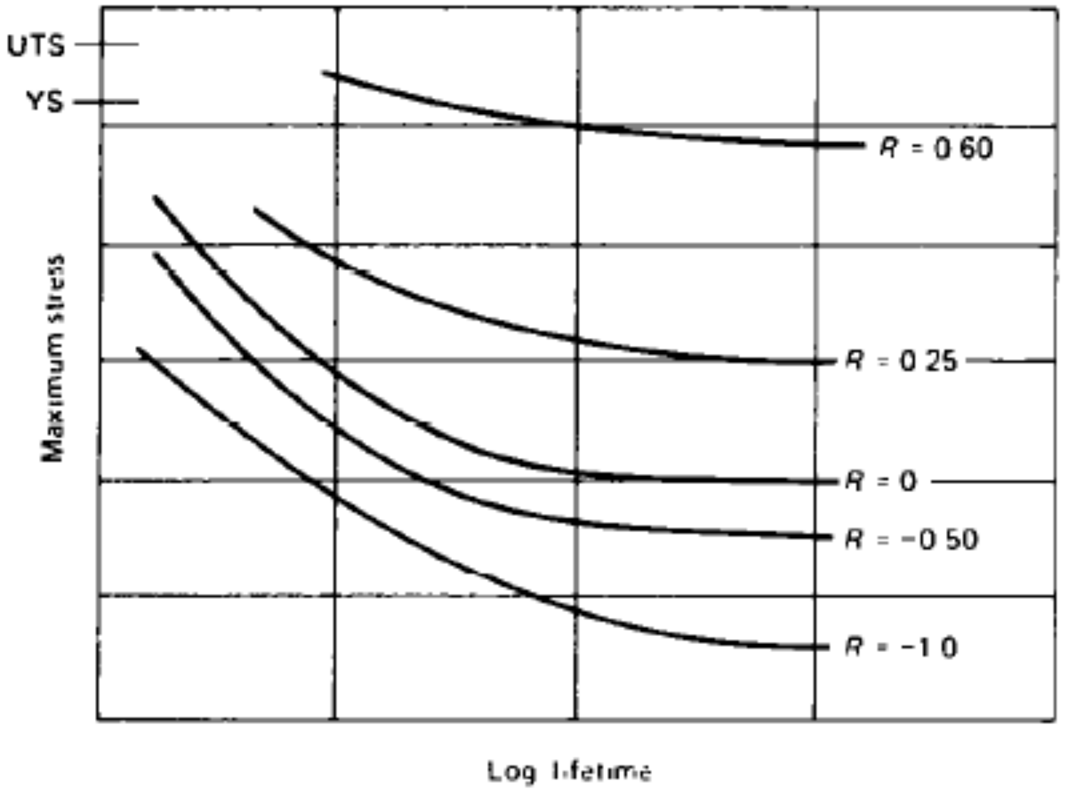




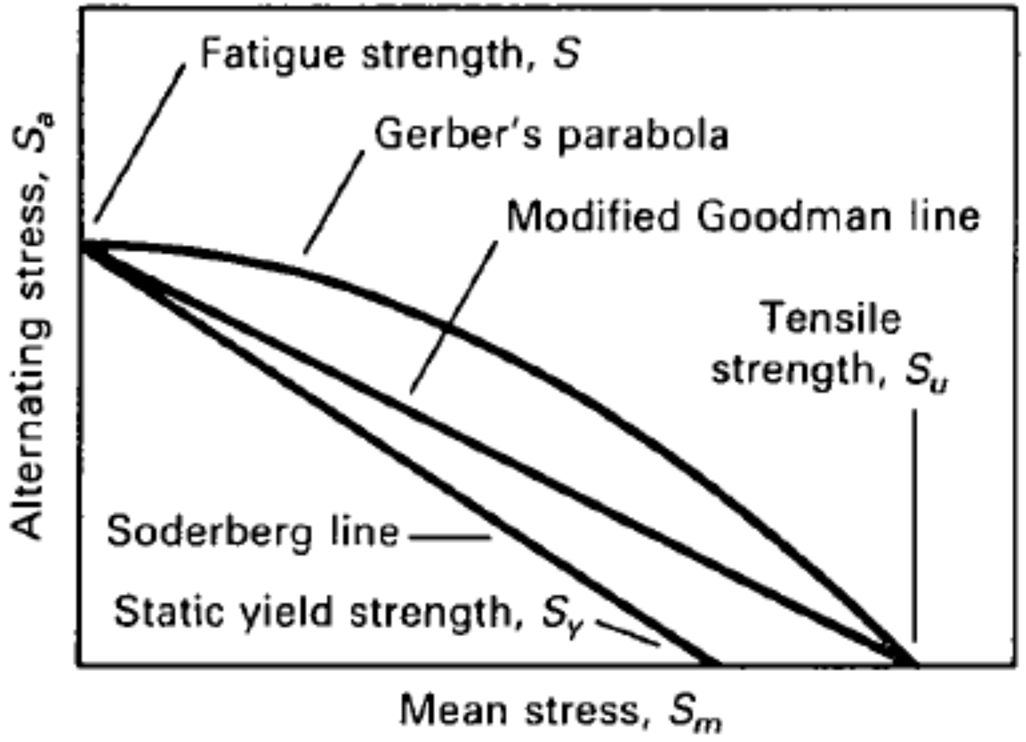
Question: How does σ_{fat} change with R?

Answer: Let’s do experiments and find out

Experiments and models

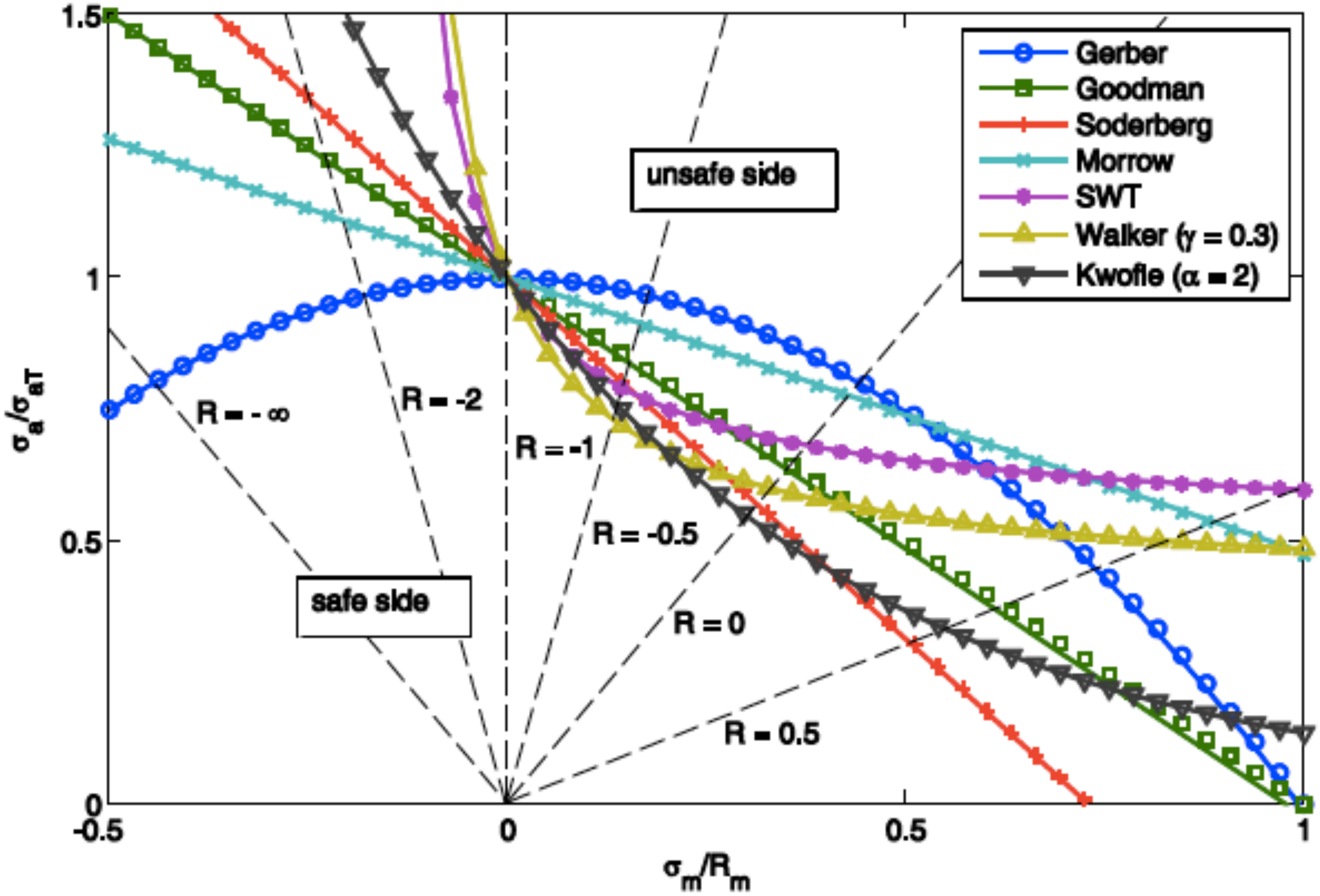


Typical S-N data for steel



Proposed models (there are more)

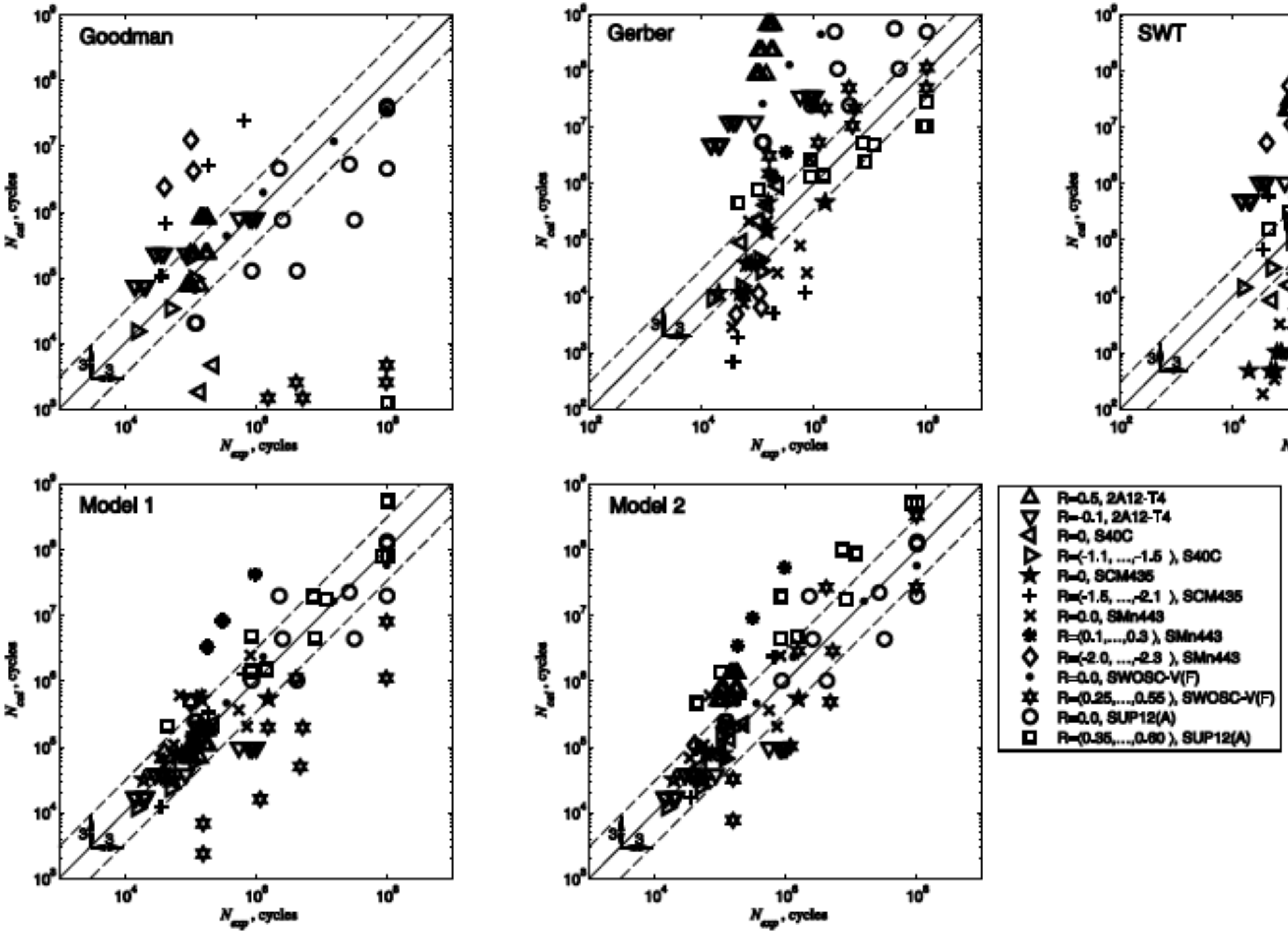
Constant Life Diagram (CLD)



Note: Widely different Predictions by models

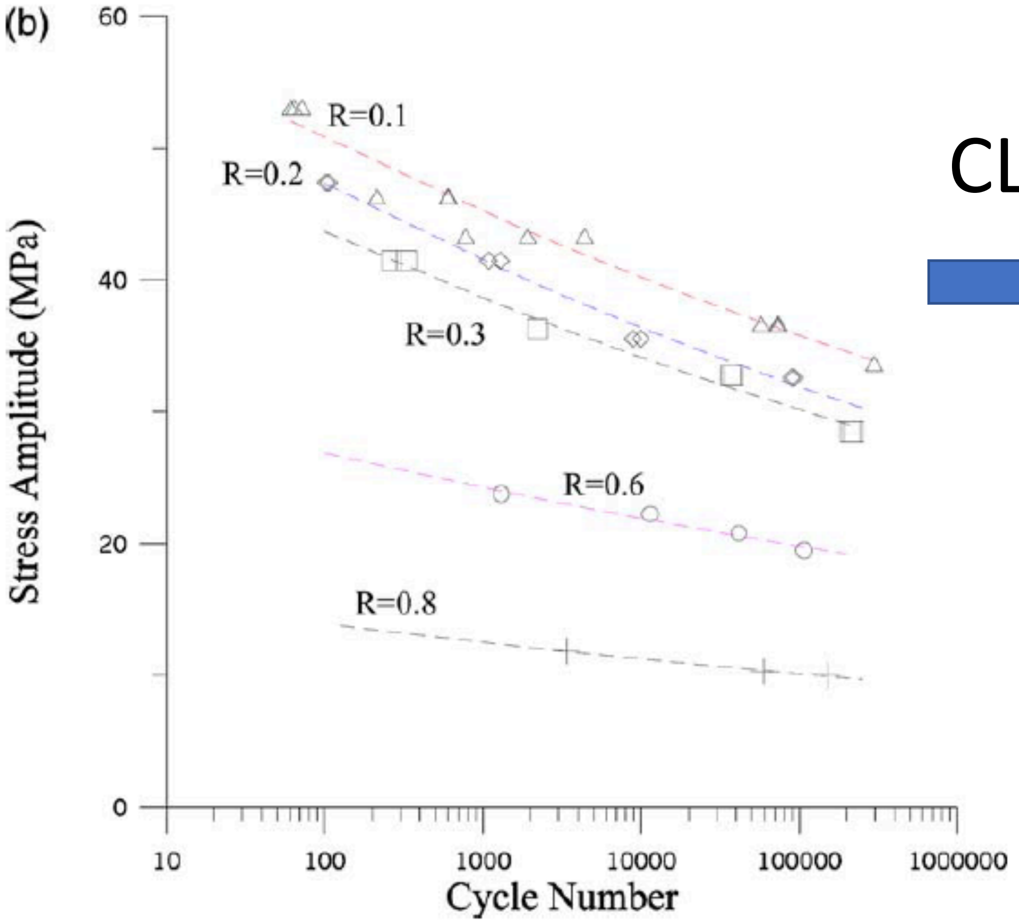
⇒ No choice but to do testing, a lot of it!

Not convinced?: Look at Comparison with experimental data – not good!

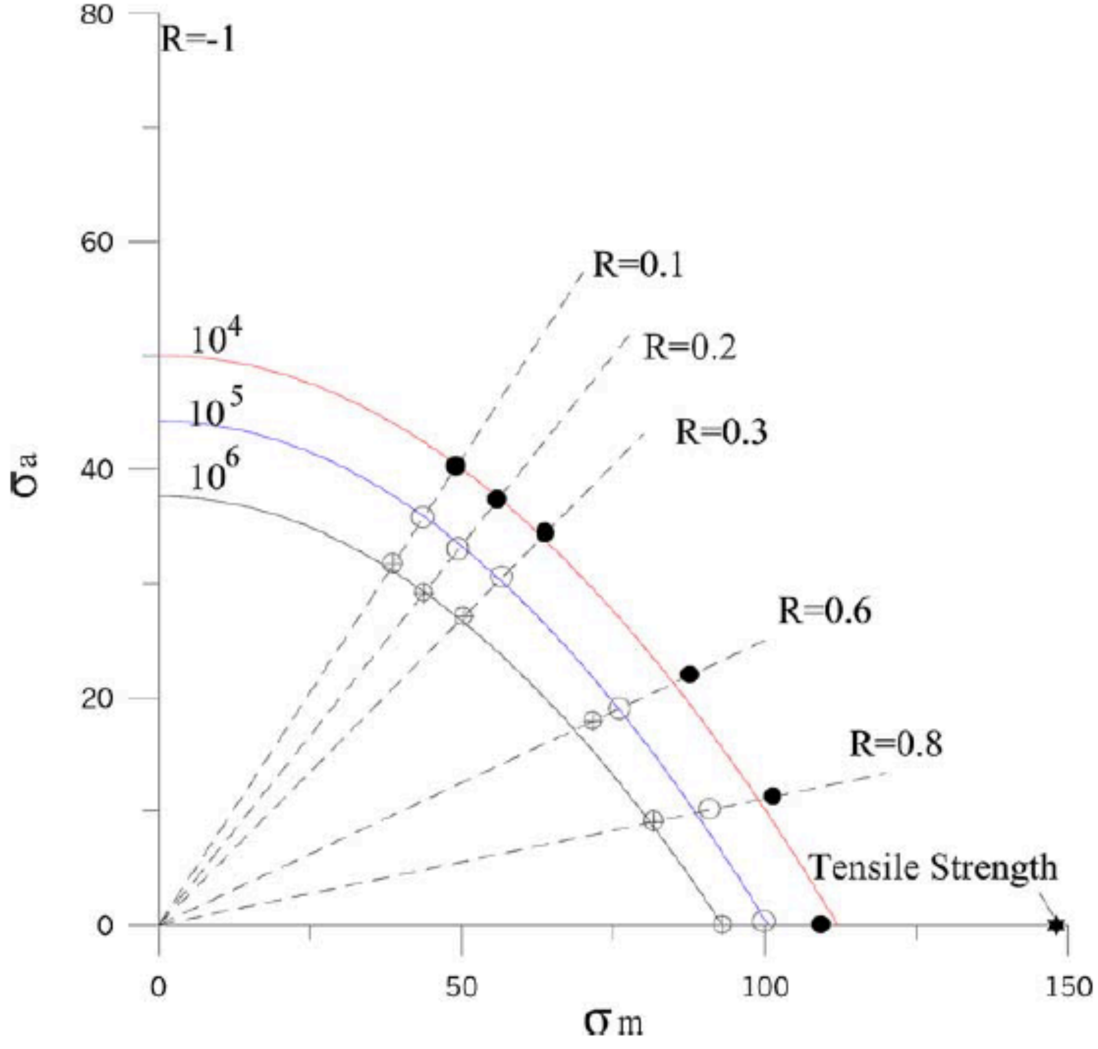


Constant life diagrams for short fiber composites (E-glass/polyamide 6,6) Data: Mallick and Zhou, IJF, 2004

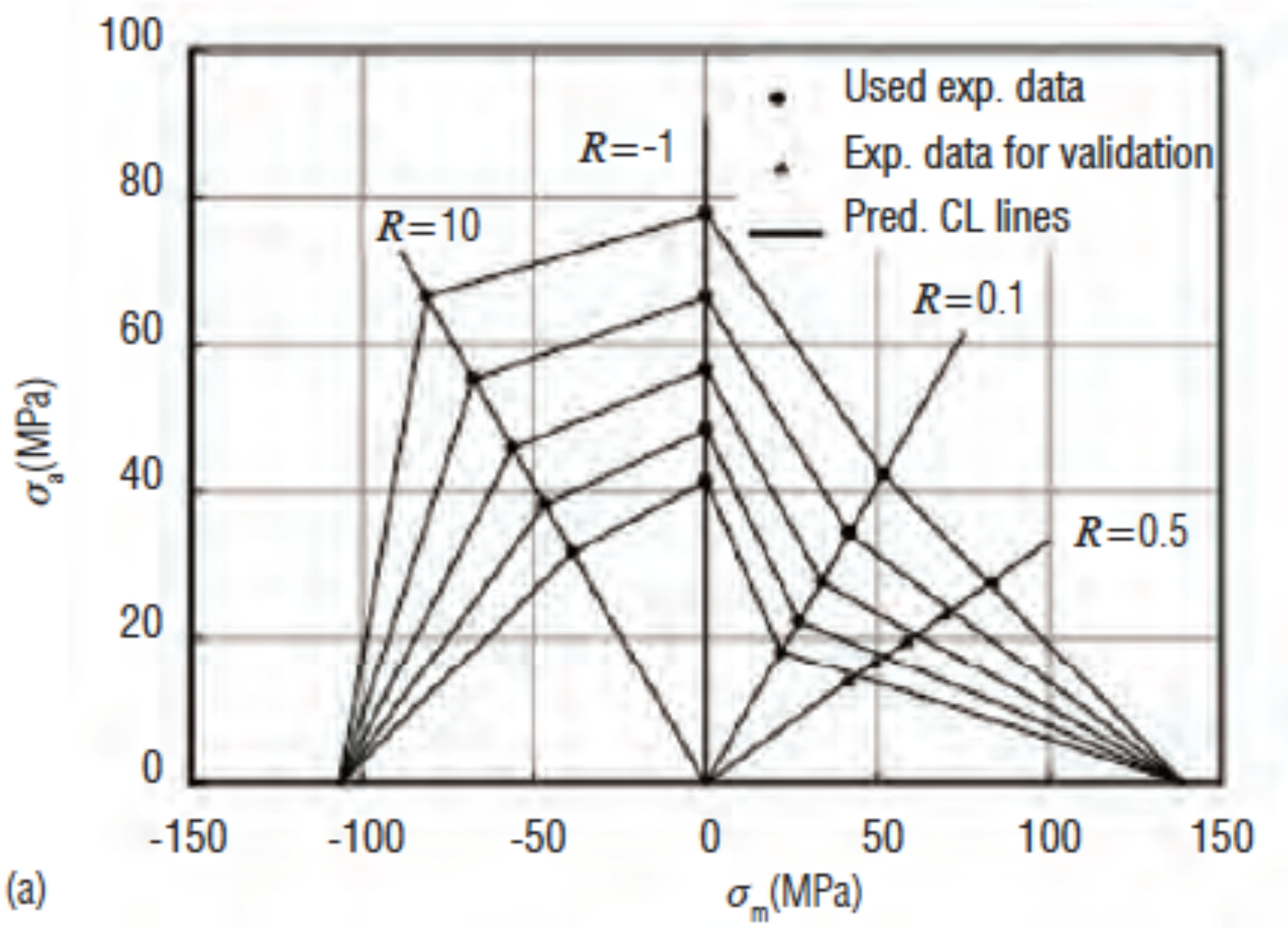
Plotting without models



CLD



Constant life diagrams for glass/epoxy (wind turbine materials) (from Nijssen, 2006)



Note:
"Predictions" sensitive to how much S-N data are used

At best, these diagrams provide overview of how fatigue life changes with mean stress (R ratio)

Note the asymmetry in tension (right of $R = -1$) vs compression (left of $R = -1$) due to mechanism differences

Phenomenological models

The ”phenomenon” of fatigue is assumed to reflect itself in some measurable property, whose rate of change (degradation) can be taken as a function of applied cyclic loading parameters.

$$\frac{d(property)}{dN} = f(\sigma_a, R, \ldots)$$

Property: strength, stiffness, dissipated energy, entropy, etc.

Phenomenological models – metal fatigue

Case 1 (Axles, thick parts without stress concentrations)

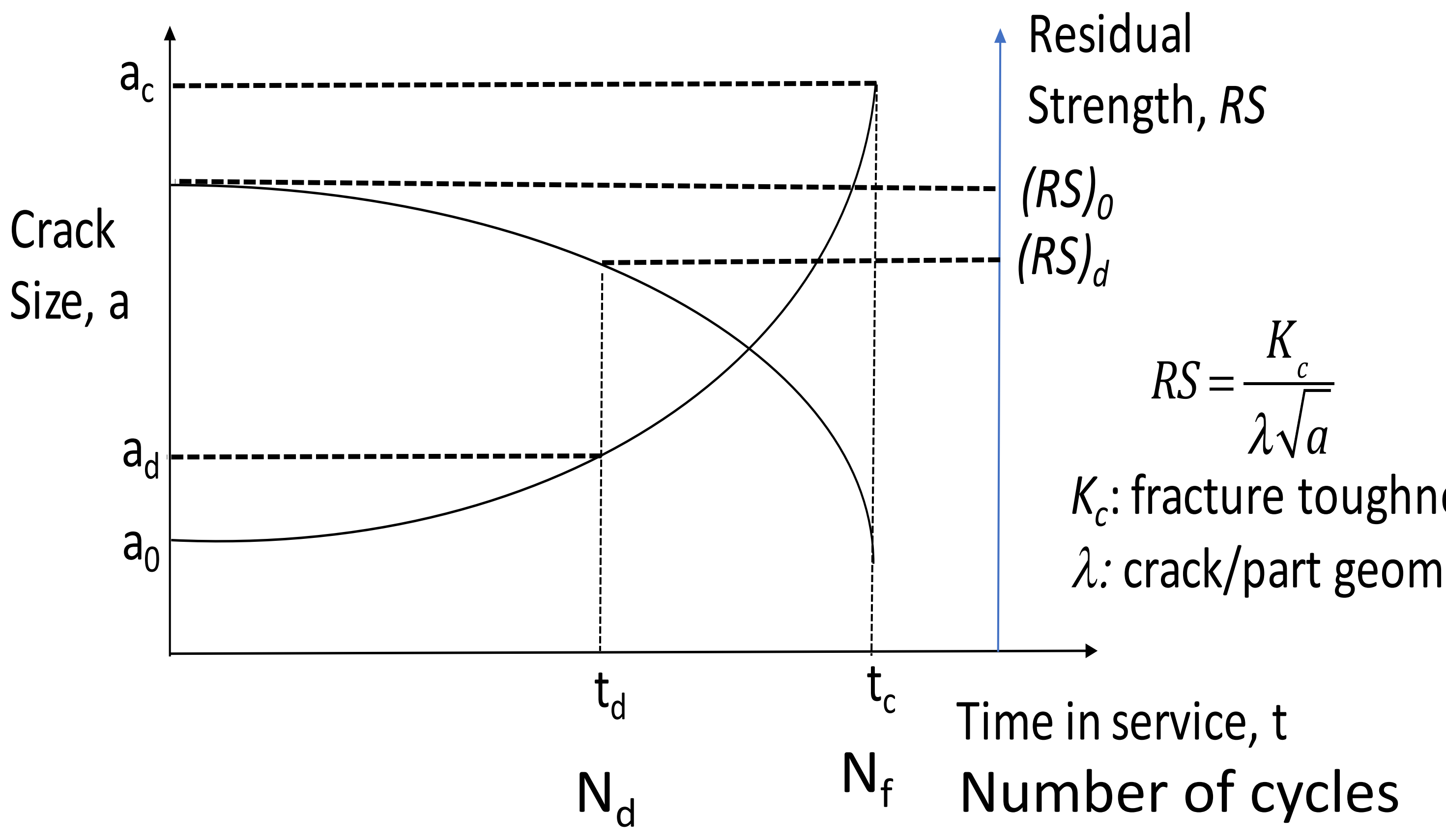
- Crack initiation dominates
- No measurable property change

Case 2 (Welded parts, sharp notches, corners, etc.)

- Crack propagation dominates
- (Residual) strength changes

Phenomenological models – metal fatigue

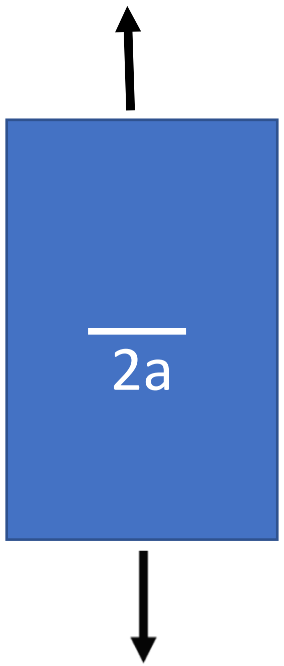
Strength degradation



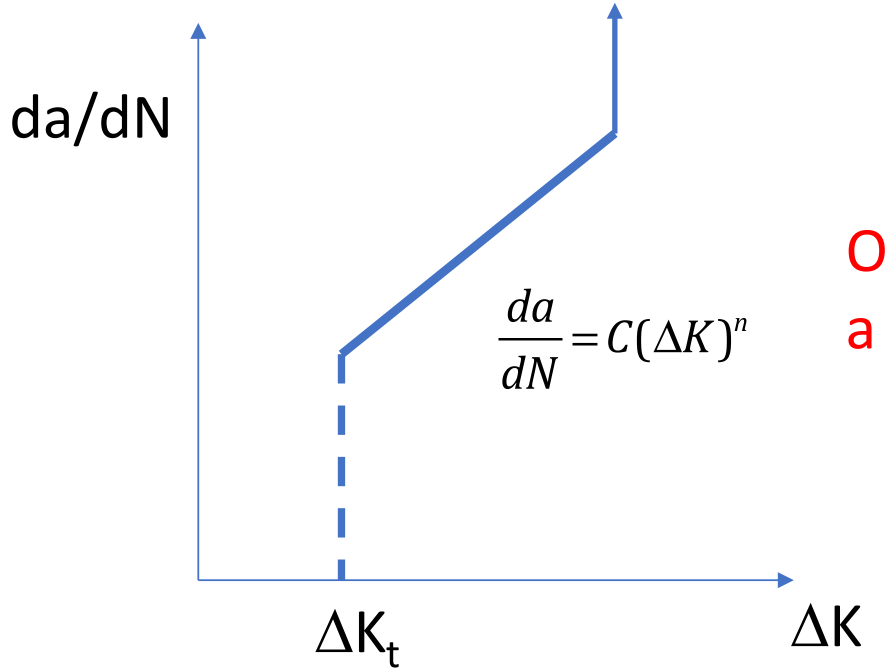
$$RS = \frac{K_c}{\lambda\sqrt{a}}$$

K_c: fracture toughness

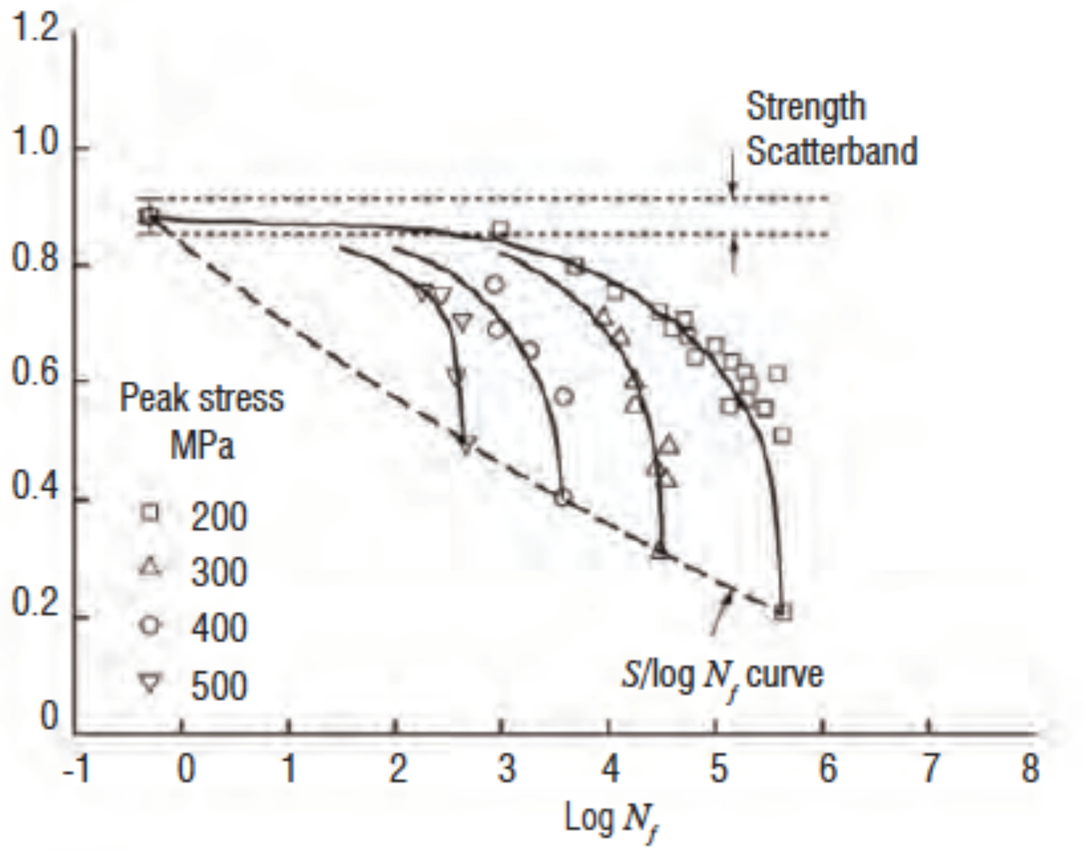
λ: crack/part geometry factor



Fatigue life prediction from strength degradation



Only applicable for a single crack.



$$\frac{da}{dN} = C(\Delta K)^n = C(\Delta\sigma\sqrt{\pi a})^n = \gamma a^{n/2}$$

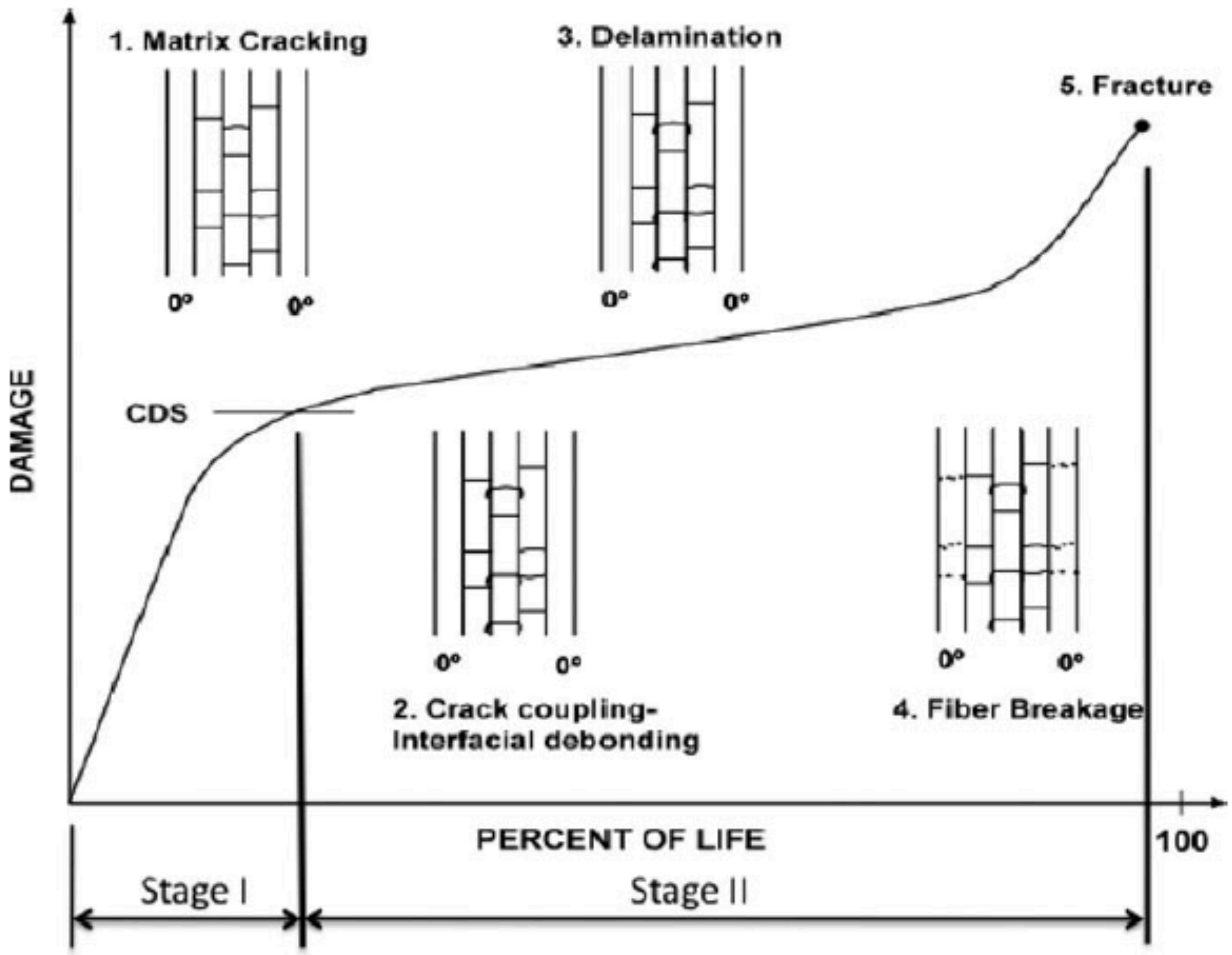
Integrate from a_0 to a_c and use $RS = \frac{K_c}{\lambda\sqrt{a}}$

=> $N_f = const(\sigma_{uts}^{n-2} - \sigma_{\max}^{n-2})$

Applying residual strength models to composite laminates

- Strictly speaking, residual strength only makes sense when ***a single pre-existing crack*** becomes ***unstable*** under applied load, i.e., ***brittle failure.*** Griffith criterion then gives $RS = \frac{K_c}{\lambda\sqrt{a}}$
- Residual strength (or strength degradation) can be assumed if general weakening (diffused cracking) occurs. Griffith criterion, however, will not apply. Diffused (distributed) cracking can be described as "damage", given by a severity index D
- Conditions for Griffith criterion do not exist in unidirectional composites. Also, general weakening does not occur.

Damage in composite laminates



"Damage" here is not quantified, but indicates qualitatively the rate at which the mechanisms are found to increase

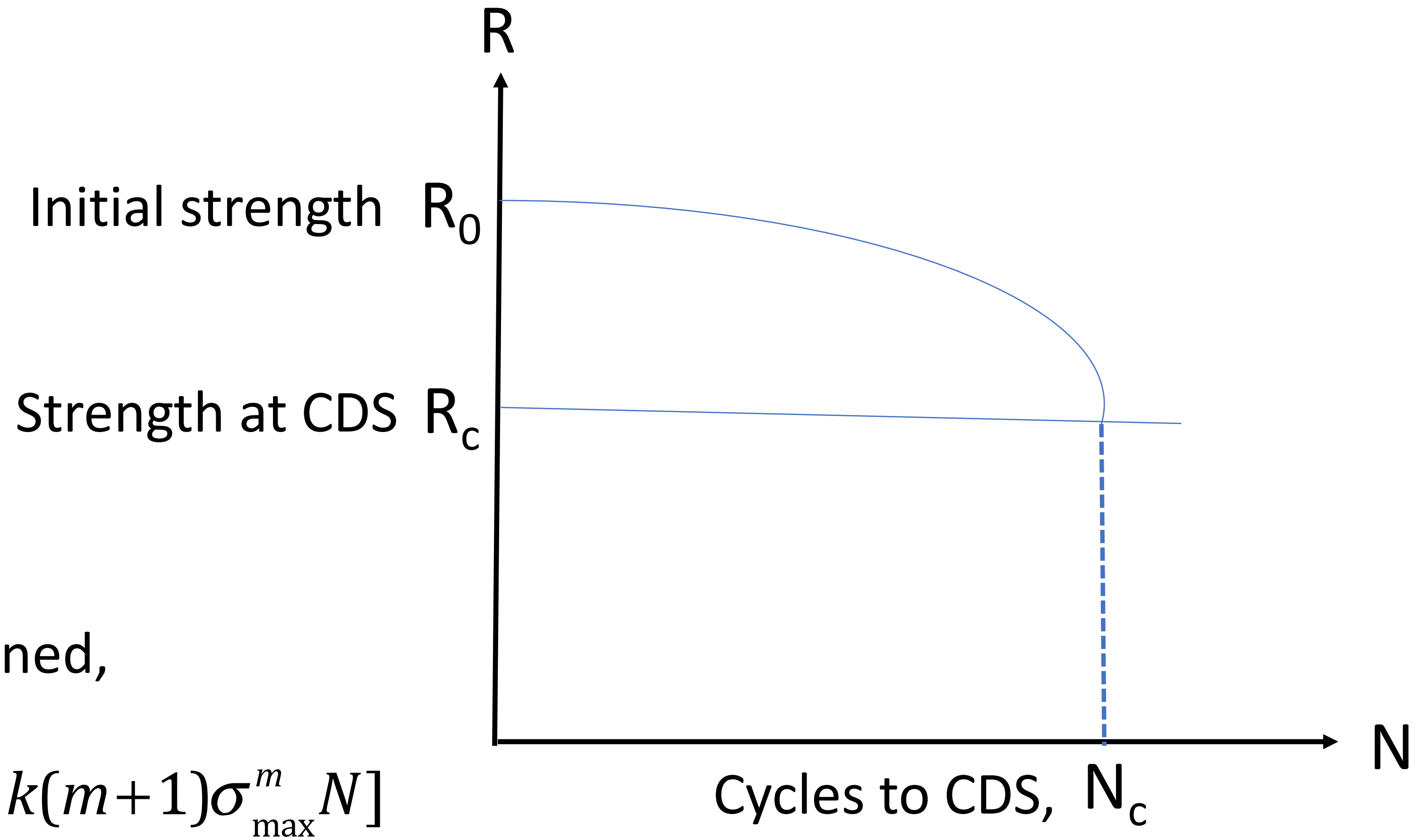
CDS: A state of saturation of matrix cracking, characteristic of a given laminate

Based on Jamison, et al, 1984

Stage I – Distributed matrix cracking

Define, $D = \frac{R_0 - R}{R_0 - R_c}$

Assume, $\frac{dD}{dN} = k\left(\frac{\sigma_{\max}}{1 - D}\right)^m$



Integrating and using D as defined,

$$(R - R_c)^{m+1} = (R_0 - R_c)^{m+1}[1 - k(m+1)\sigma_{\max}^m N]$$

R_0, R_c, m and k are to be determined from experimental data

Talreja, ZAMM, 2015

Stage II – Localized damage

Define residual strength, $R = \frac{\alpha}{\sqrt{C}}$

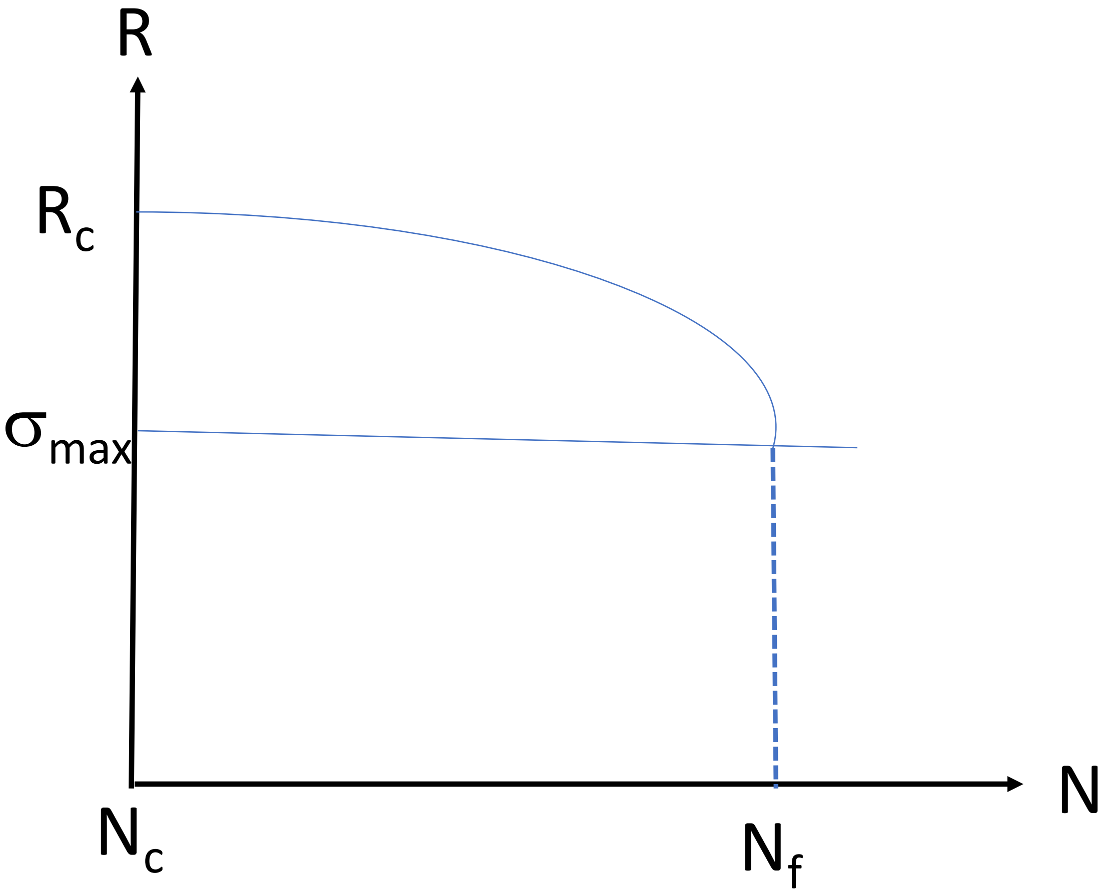
where C = "size" of localized damage
and α is similar to fracture toughness

Assume, $\frac{dC}{dN} = \gamma(C)^{n/2}$

Integrating and using R as defined,

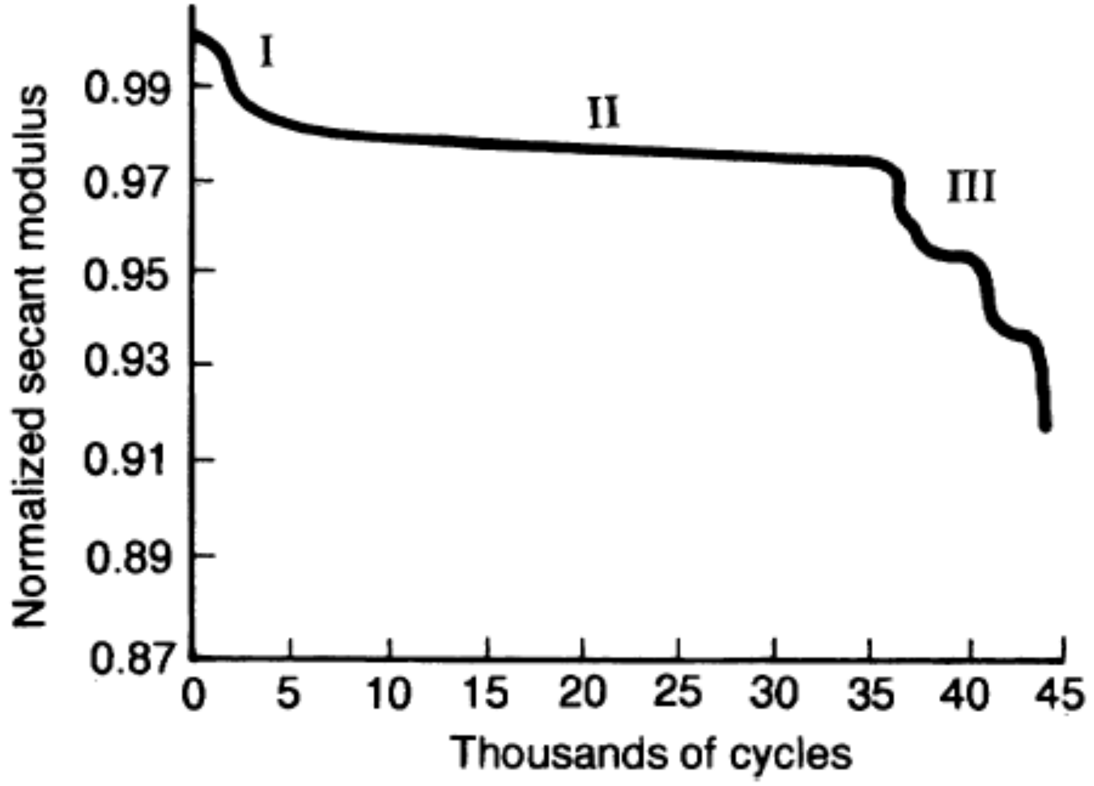
$$R_c^{n-2} - R^{n-2} = 2\gamma(n-2)(N - N_c)$$

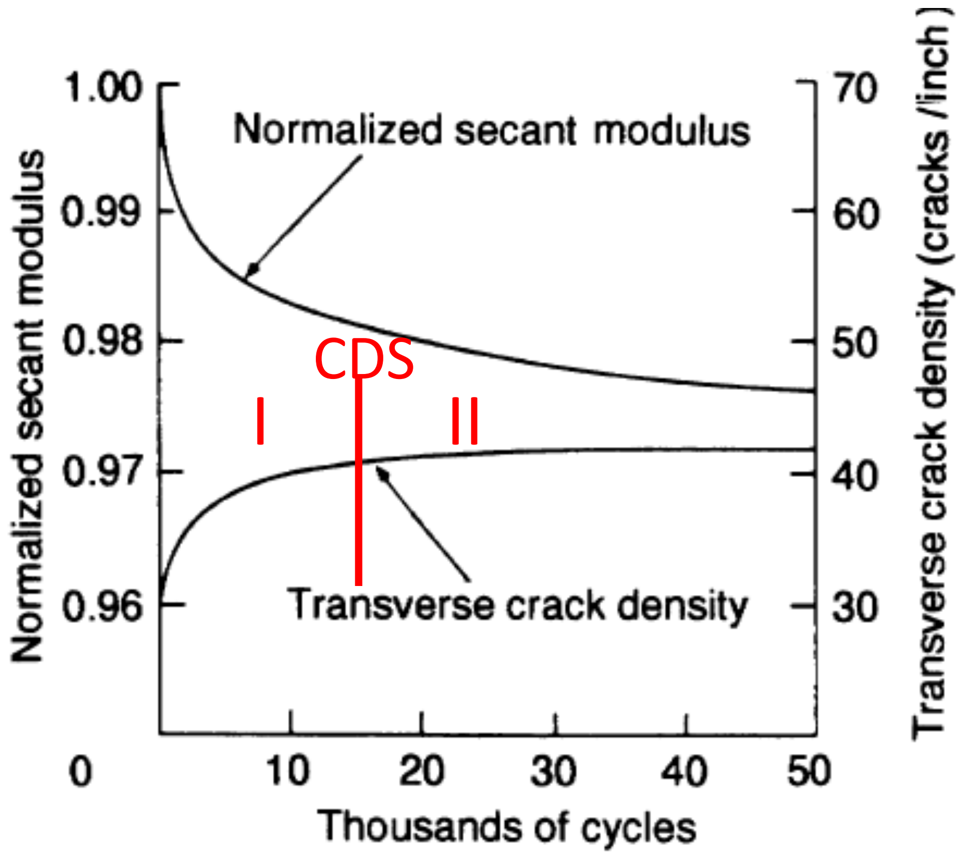
$$N_f = N_c + \left(\frac{R_c^{n-2} - \sigma_{\max}^{n-2}}{2\gamma(n-2)} \right)$$



Talreja, ZAMM, 2015

Stiffness degradation based phenomenological models – Stage I





Jamison, et al, 1984

Note: E-modulus decreases mostly in Stage I

Stiffness degradation based phenomenological models – Stage I

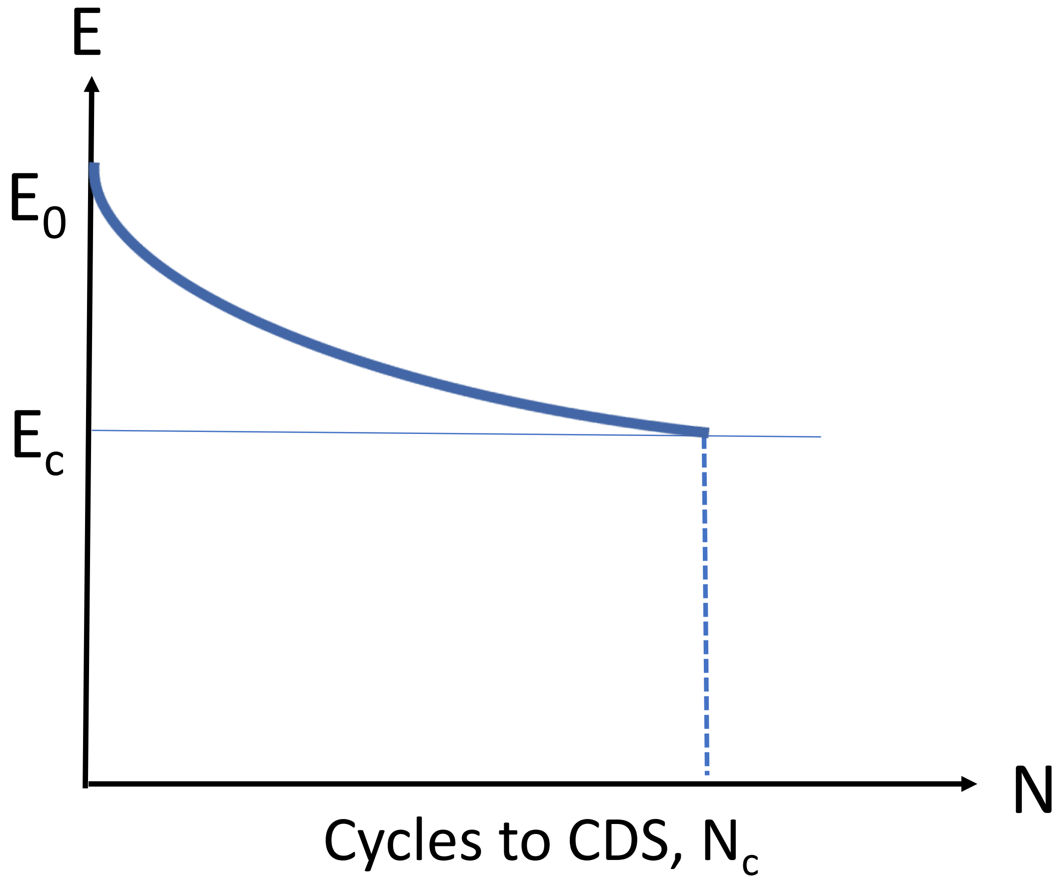
From damage mechanics theories,

$$E = E_0(1 - D)$$

=> $D = 1 - \dfrac{E}{E_0}$ Note $D \neq 1$

$$\frac{dD}{dN} = A\frac{\sigma_{\max}^b}{(1-D)^c}$$

Integrating, $N_c = \dfrac{(c+1) - (E_0/E_c)^{c+1}}{(c+1)A\sigma_{\max}^b}$

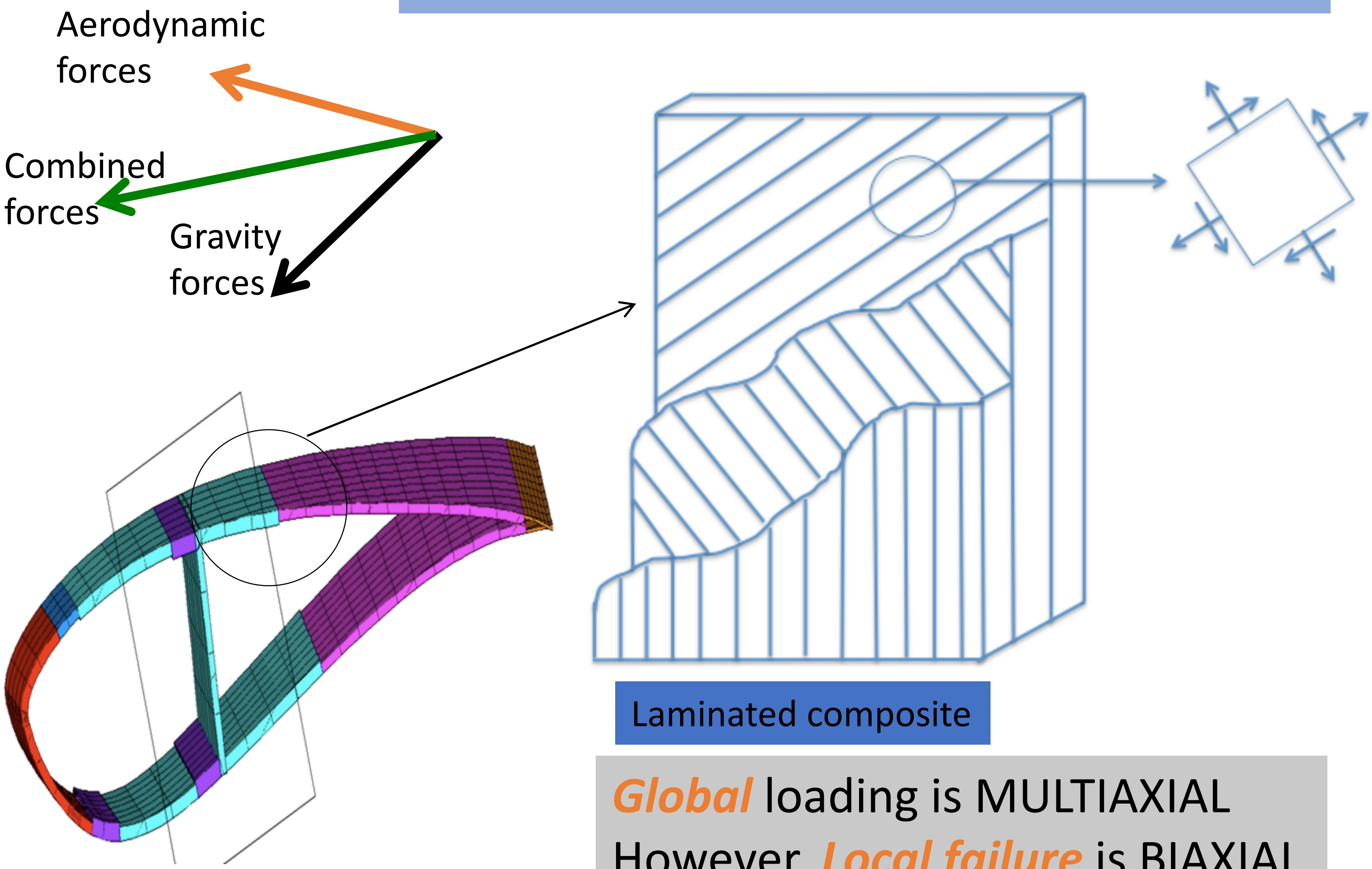


Progressive damage models

- These models depend on how “damage” is defined/characterized
- If “damage” is simply a number, e.g. $0 < D < 1$, then the models are phenomenological
- If “damage” is related to the internal energy dissipating mechanisms, and its “progression” (rate of change) is derived from those mechanisms, then the corresponding models are ***Progressive damage models*** (PDMs)

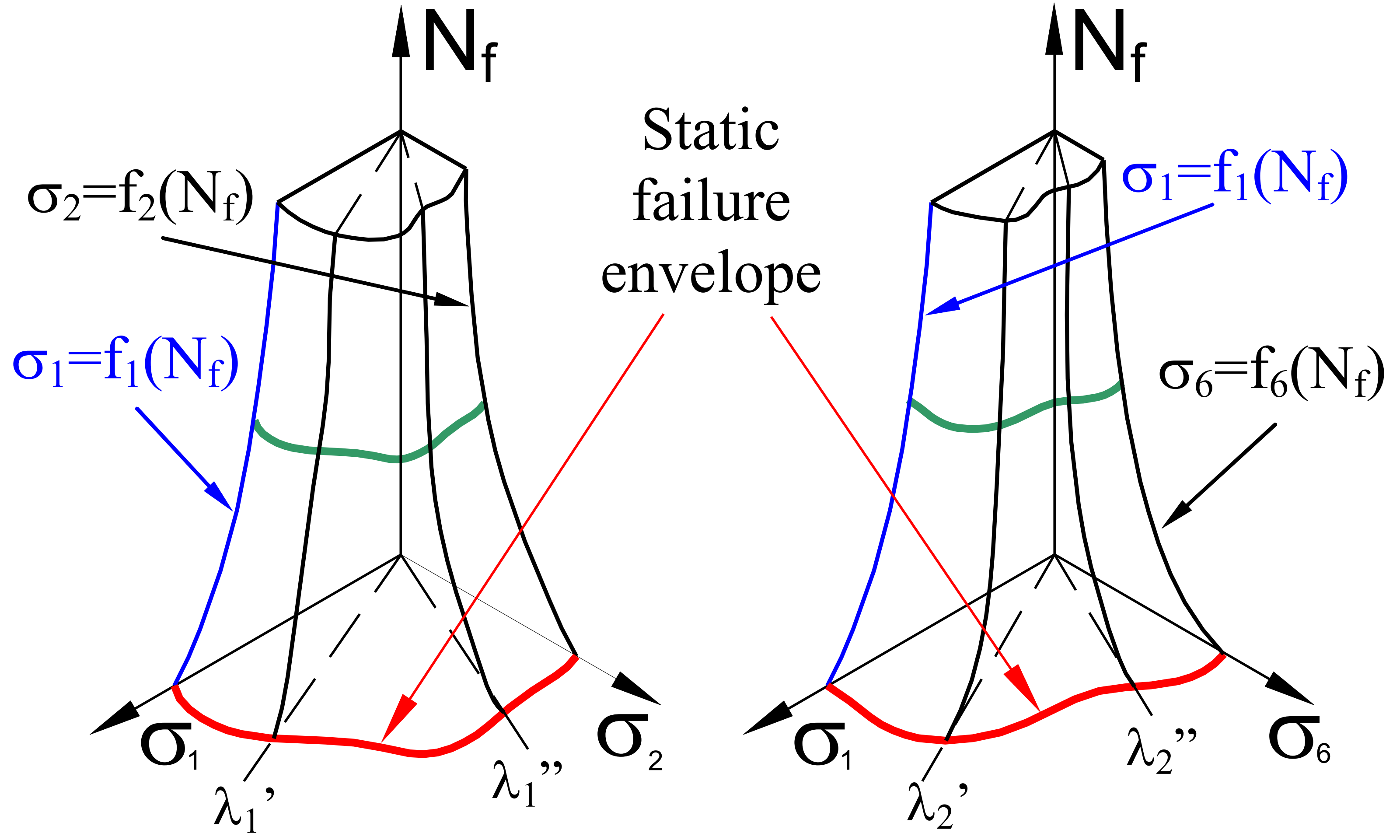
Many existing models are phenomenological but are sold as PDMs.

Failure in Multi-axial Fatigue



Global loading is MULTIAXIAL
However, *Local failure* is BIAXIAL

Multiaxial fatigue – failure criteria based approach



Assumption:
failure envelope in static loading reduces as a function of number of cycles.

Let us examine the failure criteria

Early failure theories for anisotropic solids Hill (1948), Azzi-Tsai (1965), Tsai-Wu (1971)

- Based on metal yielding
- No consideration of fiber and interface failure
- Assumed interactions of failure modes without any basis in actual physics of failure modes

Background - Metal Plasticity: Stress-based yield criteria

von Mises (1913):

Yielding initiates when the second invariant of the *deviatoric stress* tensor reaches a critical value (achieved in uniaxial stress state).

In terms of principal stresses:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 2Y^2$$

Y = yield stress

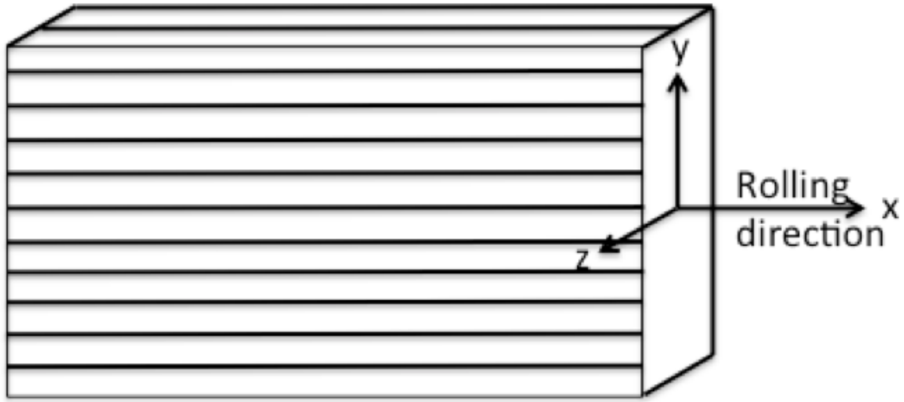
Or, in terms of stress components:

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_x - \sigma_z)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) = 2Y^2$$

Note: Hencky showed in 1923 that von Mises criterion can be derived from distortion energy

Hill's Generalization of von Mises criterion for orthotropic metals

Motivation: Texture (grain stretching in rolling direction) developed in metal sheet forming



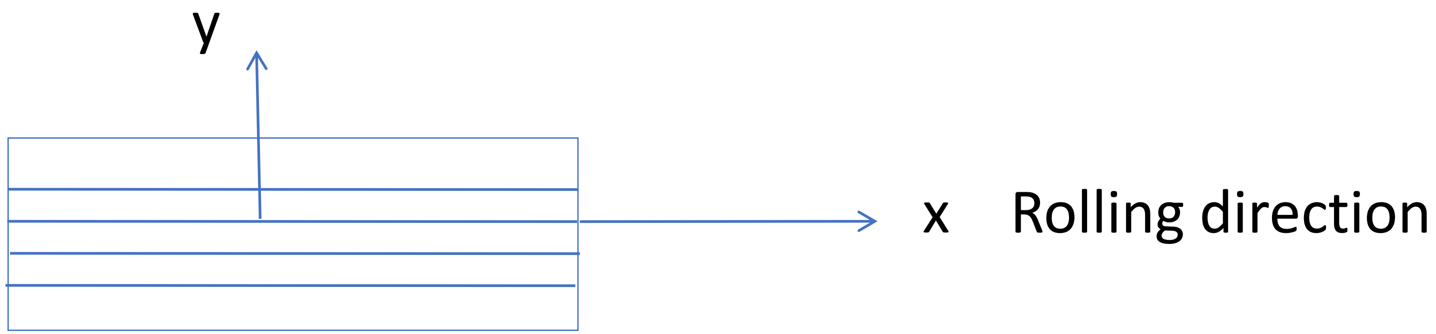
Take von Mises criterion for *isotropic* metal yielding:

$$(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_x-\sigma_z)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2)=2Y^2$$

MATHEMATICALLY Generalize it to ANISOTROPIC yielding:

$$F(\sigma_y-\sigma_z)^2+G(\sigma_z-\sigma_x)^2+H(\sigma_x-\sigma_y)^2+2L\tau_{yz}^2+2M\tau_{xz}^2+2N\tau_{xy}^2=1$$

Closer examination of Hill's Criterion



Let X, Y and Z be yield stresses in x, y and z directions
and R, S and T be yield stresses in shear in the principal planes

=>

$$\frac{1}{X^2} = G + H, \qquad 2F = \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2},$$

$$\frac{1}{Y^2} = H + F, \qquad 2G = \frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2},$$

$$\frac{1}{Z^2} = F + G, \qquad 2H = \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}.$$

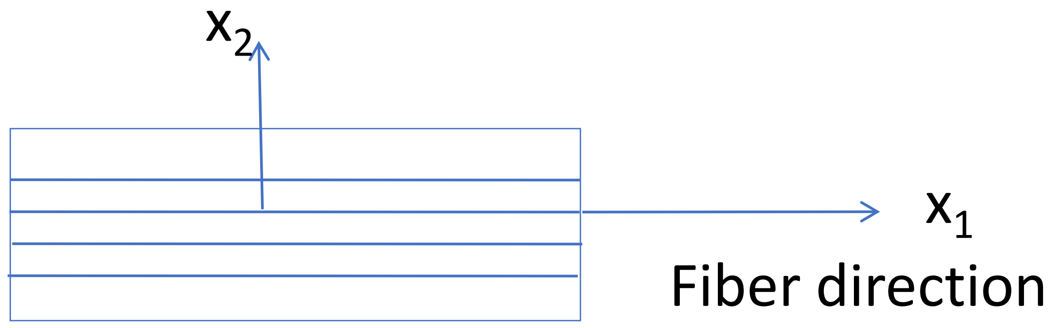
$$2L = \frac{1}{R^2}, \qquad 2M = \frac{1}{S^2}, \qquad 2N = \frac{1}{T^2}.$$

Closer examination of Hill's Criterion, contd.

- If $X = Y = Z$; $R = S = T = Y/2$, then Hill's criterion reduces to the von Mises criterion
- Hencky's interpretation of von Mises criterion as energy of distortion criterion DOES NOT apply to Hill's criterion.
- The six material constants in Hill's criterion are all for one physical phenomenon: YIELDING

Adaptation of Hill's criterion by Azzi & Tsai (1965)

REPLACE six yield stress constants with six UD composite "strength" constants. Then, in 2-D (in-plane) version, assume Y = Z (transversely isotropic).



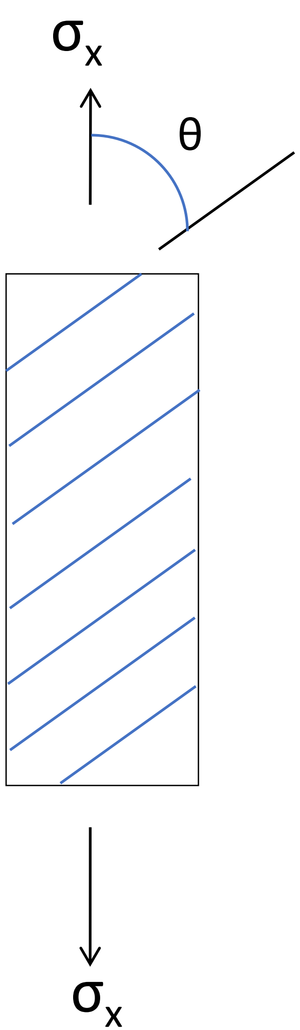
$$\left(\frac{\sigma_1}{X}\right)^2 - \left(\frac{\sigma_1 \sigma_2}{X}\right)^2 + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\tau_{12}}{T}\right)^2 = 1$$

Known as "Tsai-Hill" criterion

Tsai-Hill "Failure" Criterion - Initial Claims of success

Test case: UD composites under uniaxial load

$$\frac{\cos^4\theta}{X^2} + \left(\frac{1}{T^2} - \frac{1}{X^2}\right)\cos^2\theta\sin^2\theta + \frac{\sin^4\theta}{Y^2} = \frac{1}{\sigma_x^2}$$



- Test data seemed to agree well
- For compression, X = X', Y = Y', compressive strength values
- **Agreement for general cases not found**

Why is this not the right criterion?

Summary Remarks on Tsai-Hill and similar failure criteria

- Failure mechanisms in basic modes (tension, compression, shear) are independent, with different critical states, and cannot therefore be combined to give loss of load carrying capacity (strength).
- The governing failure mechanism for given loading condition must be identified and its criticality must be expressed in terms of combined stresses

Formal criteria with no physical basis

Gol'denblat and Kopnov (1965) proposed a "rather general" SCALAR function of TENSOR Stress Components as

$$(F_{ij}\sigma_{ij})^{\alpha} + (F_{ijkl}\sigma_{ij}\sigma_{kl})^{\beta} + (F_{ijklmn}\sigma_{ij}\sigma_{kl}\sigma_{mn})^{\gamma} = 1$$

Tsai and Wu (1971) "adapted" (simplified) this expression to an orthotropic UD composite, in Voigt notation, as

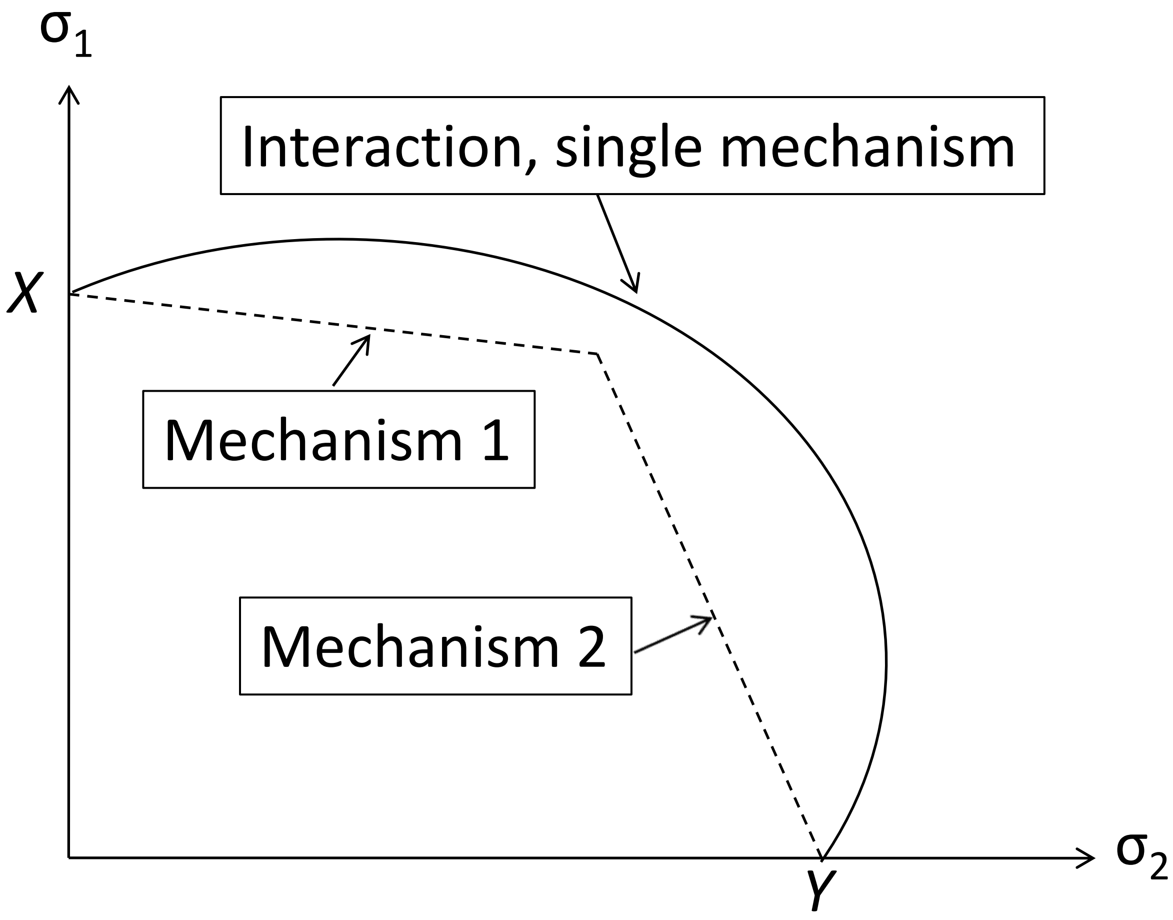
$$F_p\sigma_p + F_{pq}\sigma_p\sigma_q = 1 \quad P, q = 1, 2, 6$$

→ Failure surface: Ellipsoid or Elliptical Paraboloid

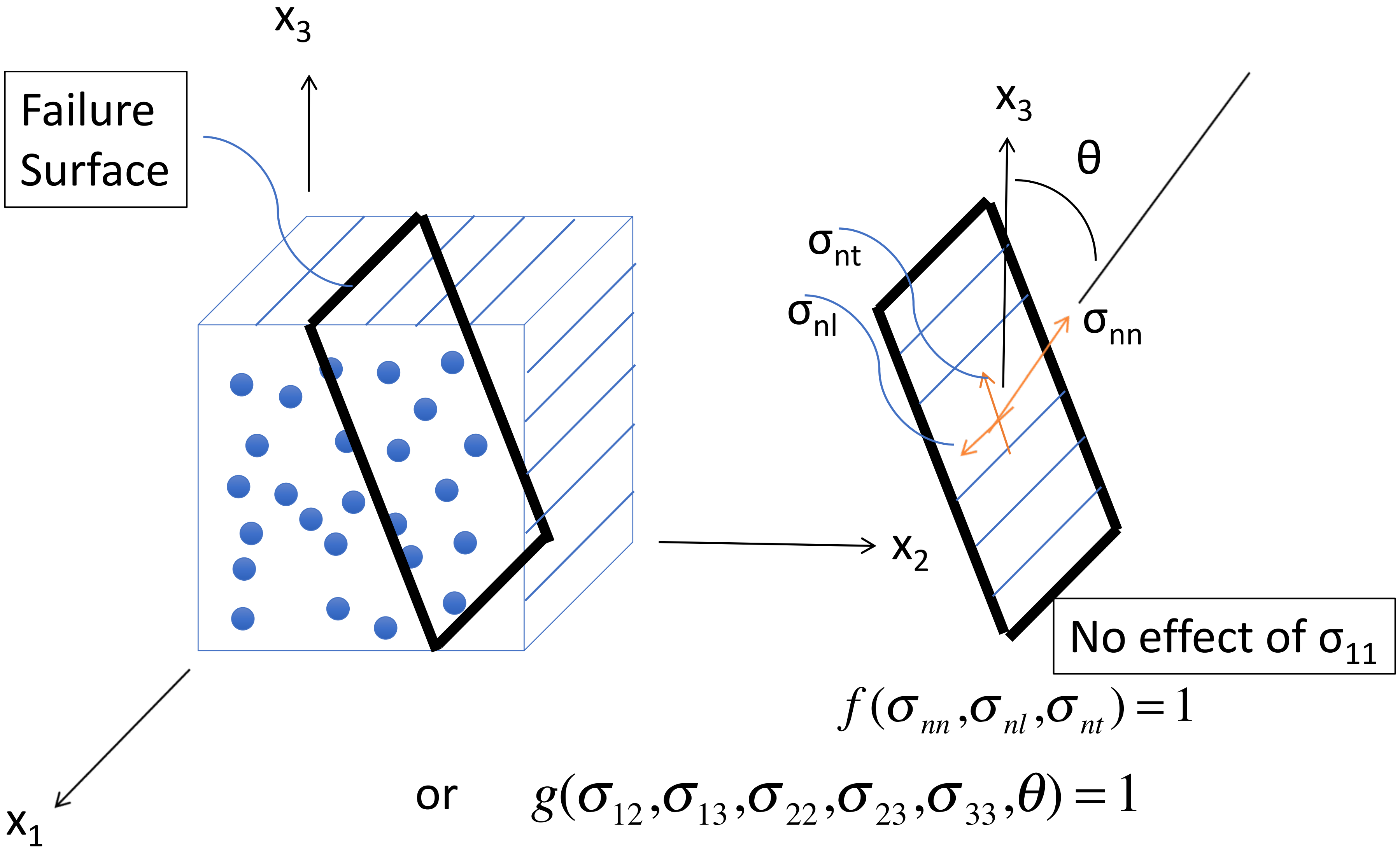
Hashin's Failure Theory:
The FIRST with some physical considerations

- Hashin (1980) observed that a single smooth (quadratic) failure surface (in σ_{11}, σ_{22}, and σ_{12}) is not supported by two independent failure mechanisms: fiber failure and matrix failure
- He proposed separate criteria for tensile fiber mode, compressive fiber mode, tensile matrix mode and compressive matrix mode
- For matrix failure modes he postulated a failure plane inspired by Coulomb's work in 1776

Failure envelope: Single vs. Multiple mechanisms



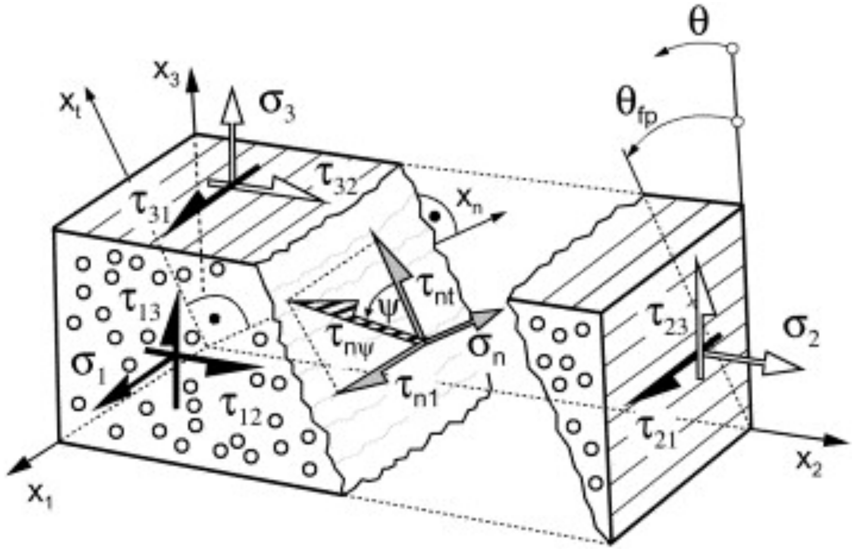
Hashin (1980) proposal for matrix failure modes



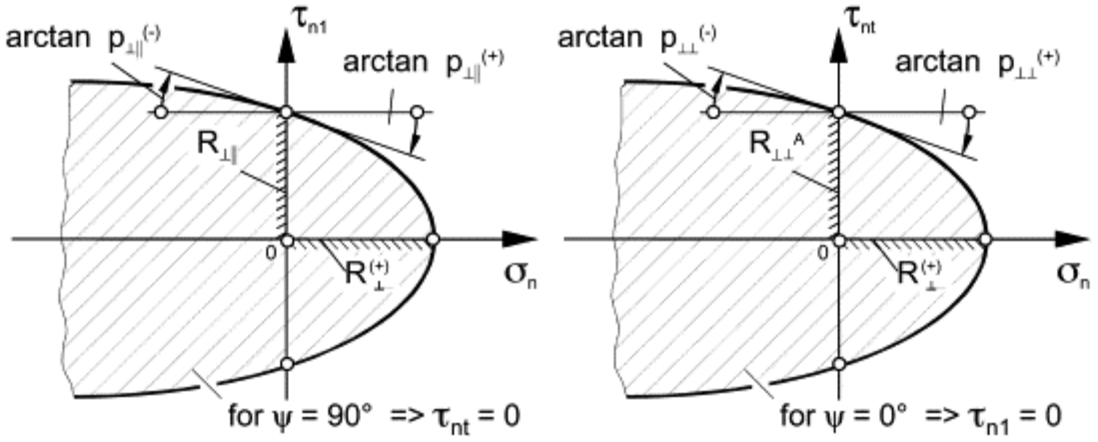
$$f(\sigma_{nn}, \sigma_{nl}, \sigma_{nt}) = 1$$

or $$g(\sigma_{12}, \sigma_{13}, \sigma_{22}, \sigma_{23}, \sigma_{33}, \theta) = 1$$

Puck’s Theory (1996):
Same assumptions as in Hashin (1980)



Seven constants needed for inter-fiber (matrix) failure mode

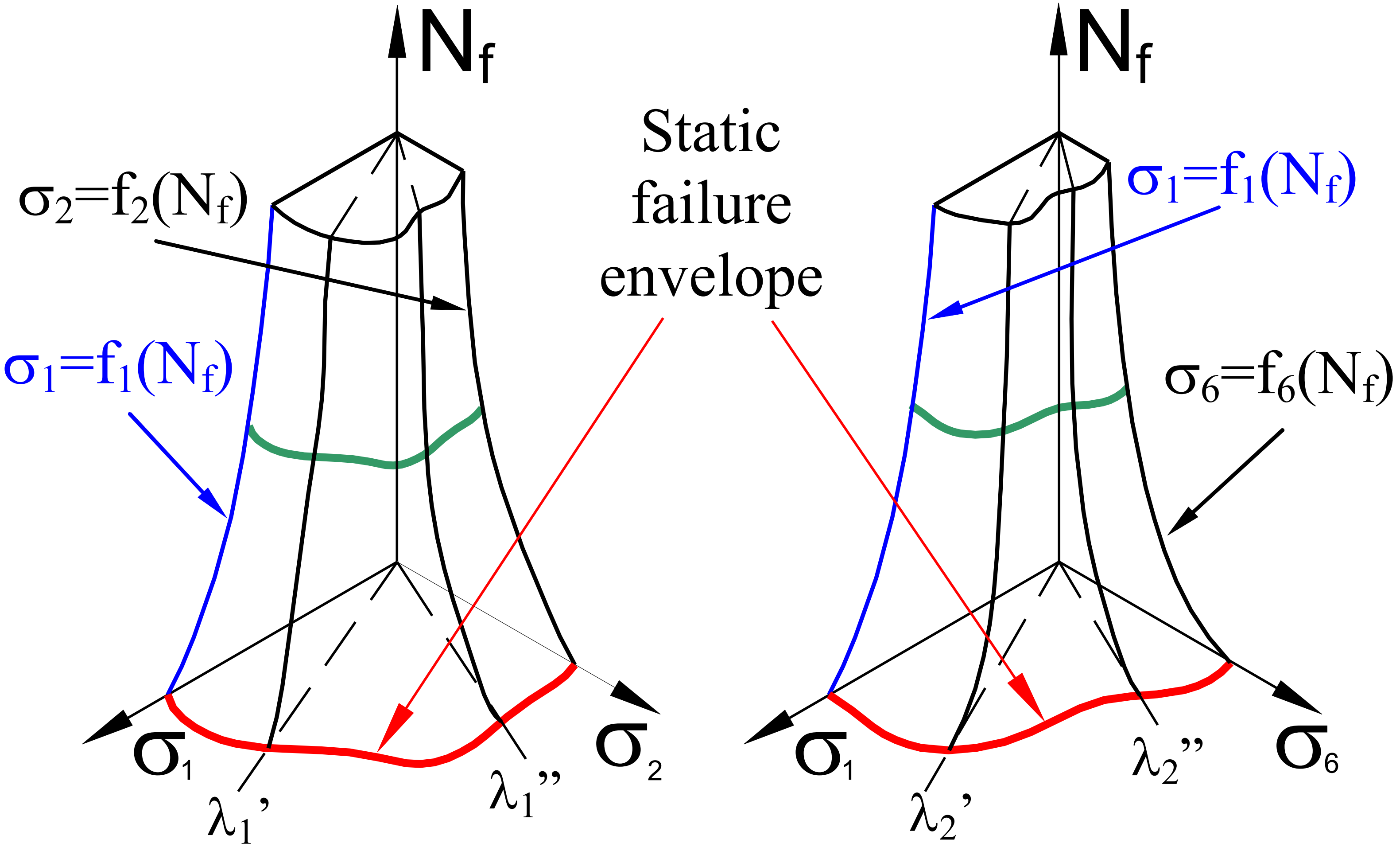


Summary Remarks on all major failure criteria

- The early criteria (Tsai-Hill, Hoffman, Tsai-Wu,..) lack physical basis, and should be regarded as obsolete.
- Hashin, followed by Puck, have physical considerations, but not direct physical evidence to support the hypotheses.

=> Need mechanisms based failure criteria.

Multiaxial fatigue – failure criteria based approach



Assumption:
failure envelope in static loading reduces as a function of number of cycles.

Let us examine the failure criteria

Cyclic biaxial stress variation

$$\sigma_x(t) = \sigma_{x,m} + \sigma_{\mathrm{x,a}} \sin(\omega \cdot t)$$

$$\sigma_y(t) = \sigma_{y,m} + \sigma_{y,a} \sin(\omega \cdot t - \delta_{y,x})$$

$$\tau_{xy}(t) = \tau_{xy,m} + \tau_{xy,a} \sin(\omega \cdot t - \delta_{xy,x})$$

In global (structural) coordinates

$$\sigma_1(t) = \sigma_{1,m} + \sigma_{\mathrm{1,a}} \sin(\omega \cdot t)$$

$$\sigma_2(t) = \sigma_{2,m} + \sigma_{2,a} \sin(\omega \cdot t - \delta_{2,1})$$

$$\sigma_6(t) = \sigma_{6,m} + \sigma_{6,a} \sin(\omega \cdot t - \delta_{6,1})$$

In local (material) coordinates

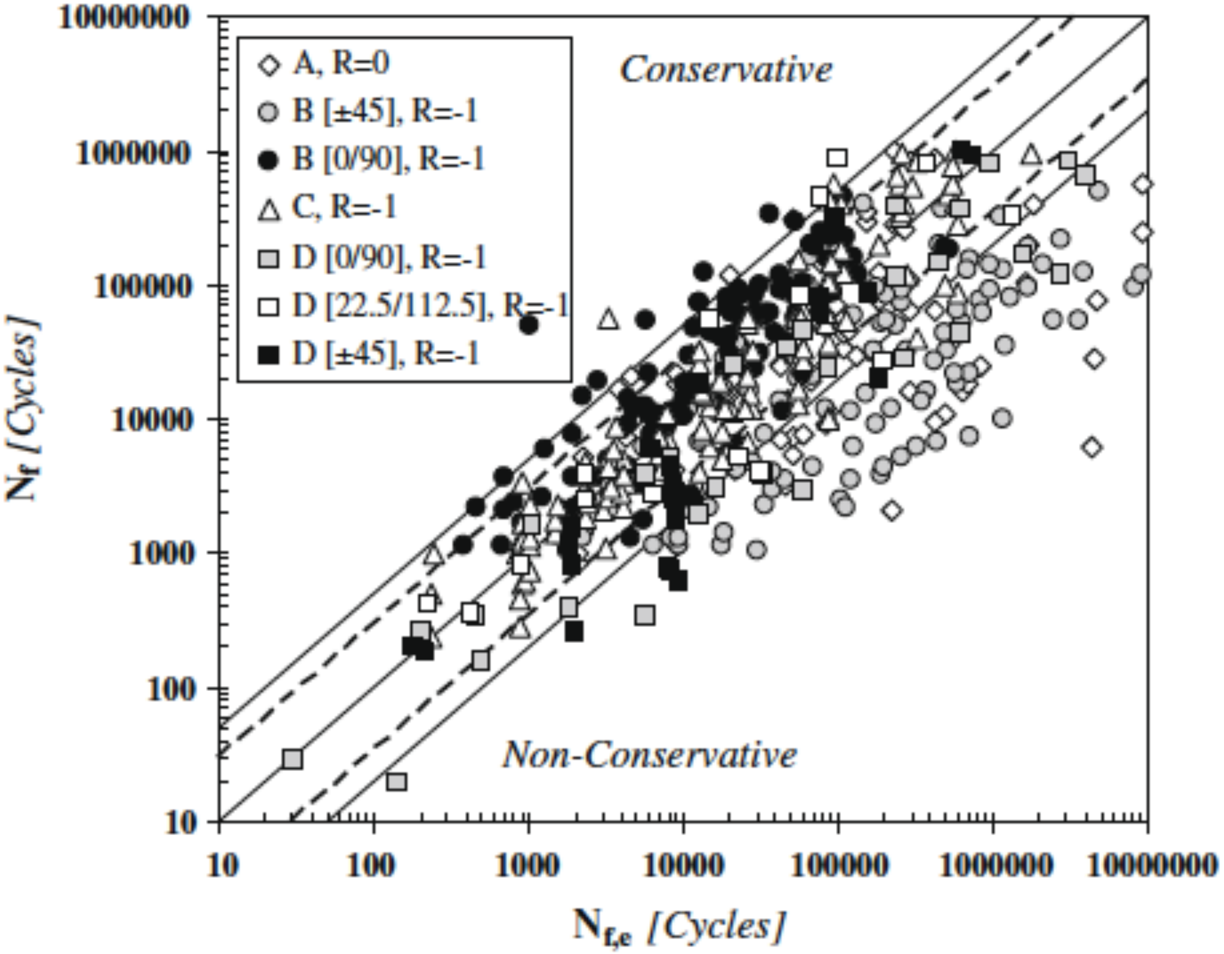
Tsai-Hill criterion based prediction of fatigue life

$$\left(\frac{\sigma_{1,a}}{K_1(N_f)}\right)^2+\left(\frac{\sigma_{2,a}}{K_2(N_f)}\right)^2-\frac{\sigma_{1,a}\sigma_{2,a}}{K_1^2(N_f)}+\left(\frac{\sigma_{6,a}}{K_6(N_f)}\right)^2=1$$

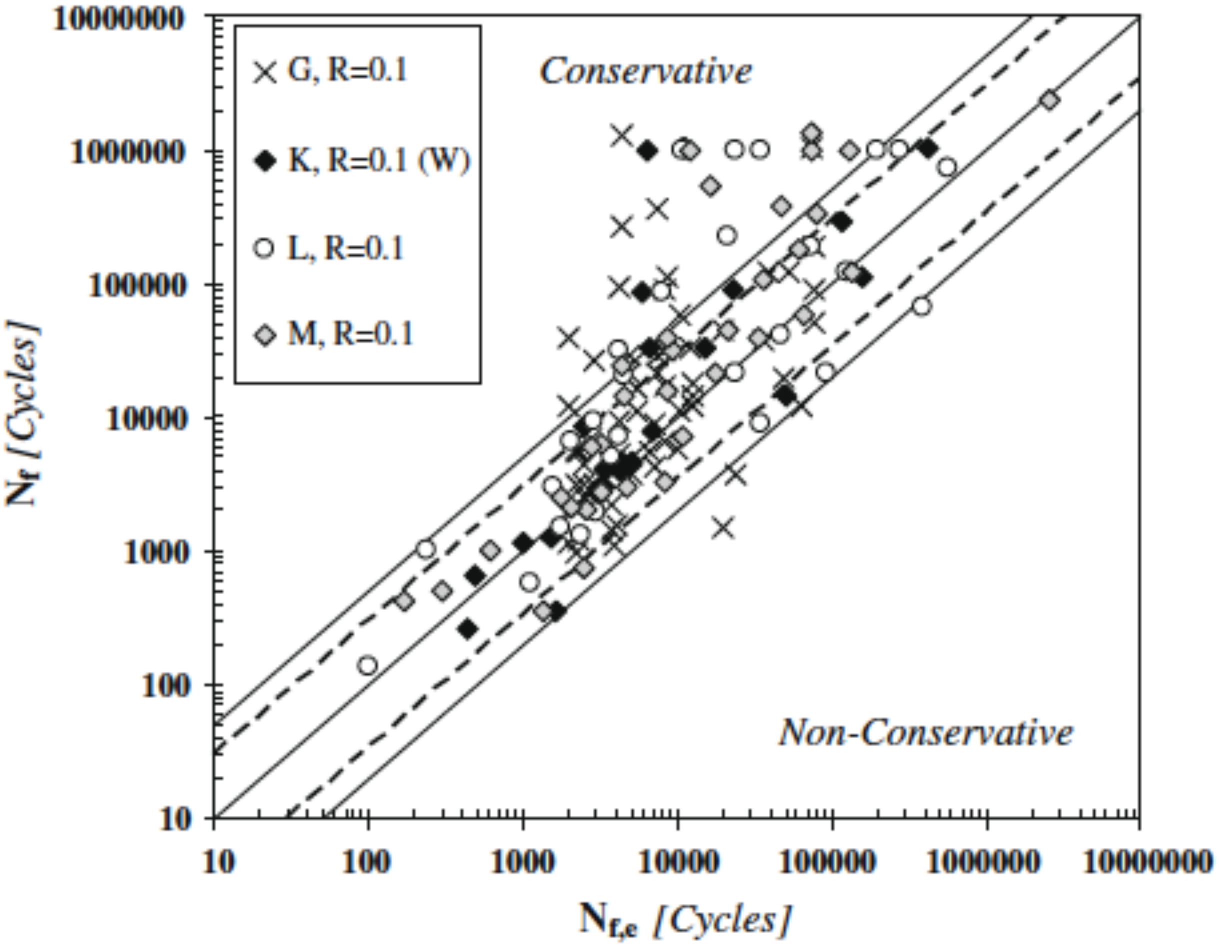
where $$K_i(N_f)=\sigma_{1,A}\left(\frac{N_A}{N_f}\right)^{1/k_i}$$

$K_i(N_f)$ have to be calculated by using experimental fatigue curves, for i = 1, 2 and 6, for the same R value.

Prediction vs Experimental data using Tsai-Hill criterion: Global coordinates



Prediction vs Experimental data using Tsai-Hill criterion: Local coordinates



Smith-Pascoe Biaxial Fatigue Criterion

Biaxial fatigue of glass-fibre reinforced composite. Part 2: failure criteria for fatigue and fracture"
Proc. of Biaxial and Multiaxial Fatigue - EGF 3 1989

1. Failure by rectilinear cracking
- governed by strain energy of normal stresses

$$U_{F,a} = \frac{1}{2}\left\{\frac{\sigma_{1,a}^2}{E_1} - \left(\frac{\nu_{12}}{E_1} + \frac{\nu_{21}}{E_2}\right)\sigma_{1,a}\sigma_{2,a} + \frac{\sigma_{2,a}^2}{E_1}\right\} = K_{SE}(N_f) = U_{F,A}\left(\frac{N_A}{N_f}\right)^{\frac{1}{k}}$$

2. Failure by shear
- governed by shear stress

$$\sigma_{6,a} = K_6(N_f) = \sigma_{6,A}\left(\frac{N_A}{N_f}\right)^{\frac{1}{k_6}}$$

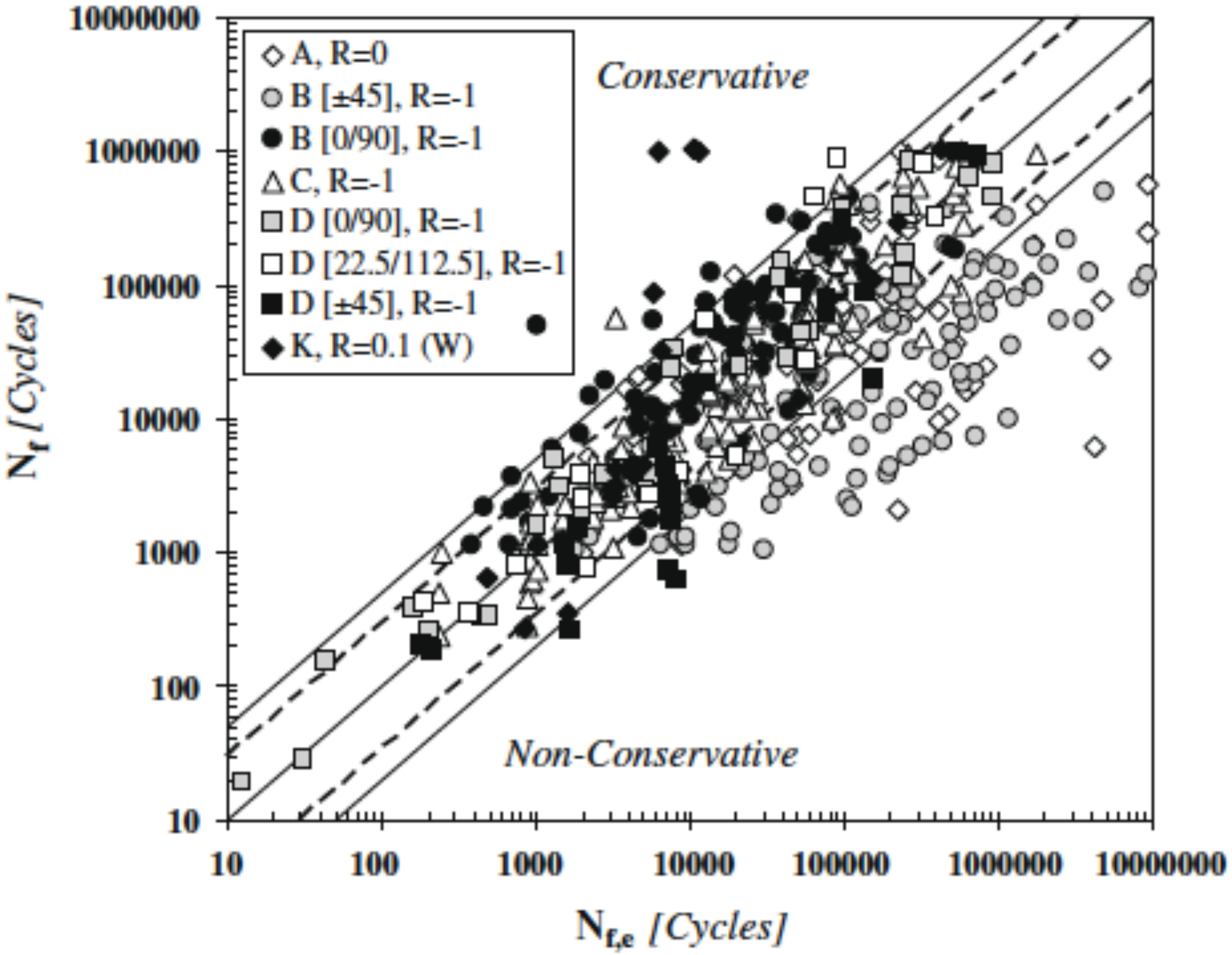
Smith-Pascoe Biaxial Fatigue Criterion

Failure when both mechanisms are present, given by

$$\frac{1}{\sigma_{1,a}^{2}(N_f)} = \frac{1}{\sigma_{SE,a}^{2}(N_f)} + \frac{1}{\sigma_{6,a}^{2}(N_f)}$$

Note:
Needs two fatigue curves for calibration, one in biaxial normal stresses, and the other in pure shear.

Prediction vs Experimental data using the Smith-Pascoe Criterion



Fewaz-Ellyin Biaxial Fatigue Criterion

"Fatigue failure model for fibre-reinforced materials under general loading conditions"
J. Composites Materials 1994; 28 (15):1432-1451.

Assumption:
If fatigue life is evaluated in one reference direction,
it can be obtained in any other direction by a transfer function.

Fatigue life in reference direction *r* is given by

$$\sigma_{r,max} = b_r + m_r \log(N_f)$$

And in any generic direction *i*,

$$\sigma_{i,max} = b_i + m_i \log(N_f)$$

Fewaz-Ellyin Biaxial Fatigue Criterion

Transfer functions:

$$m_i = f(\lambda_C, \lambda_T, \theta) \cdot g(R) \cdot m_r$$
$$b_i = f(\lambda_C, \lambda_T, \theta) \cdot b_r$$

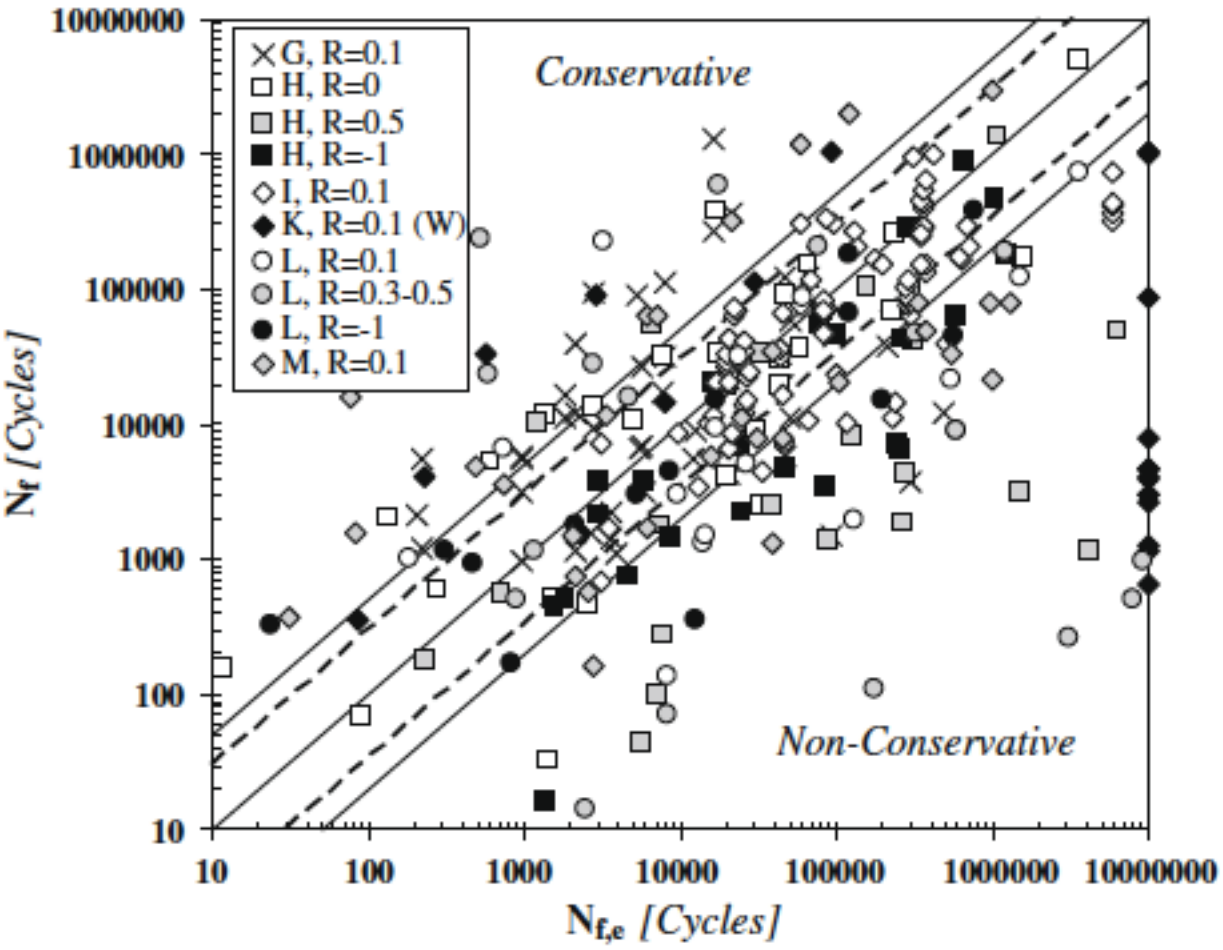
where

$$f(\lambda_C, \lambda_T, \theta) = \frac{\sigma_{x,\text{ult}}(\lambda_C, \lambda_T, \theta)}{\sigma_{x,\text{ult}}(\lambda_{C,r}, \lambda_{T,r}, \theta_r)}$$

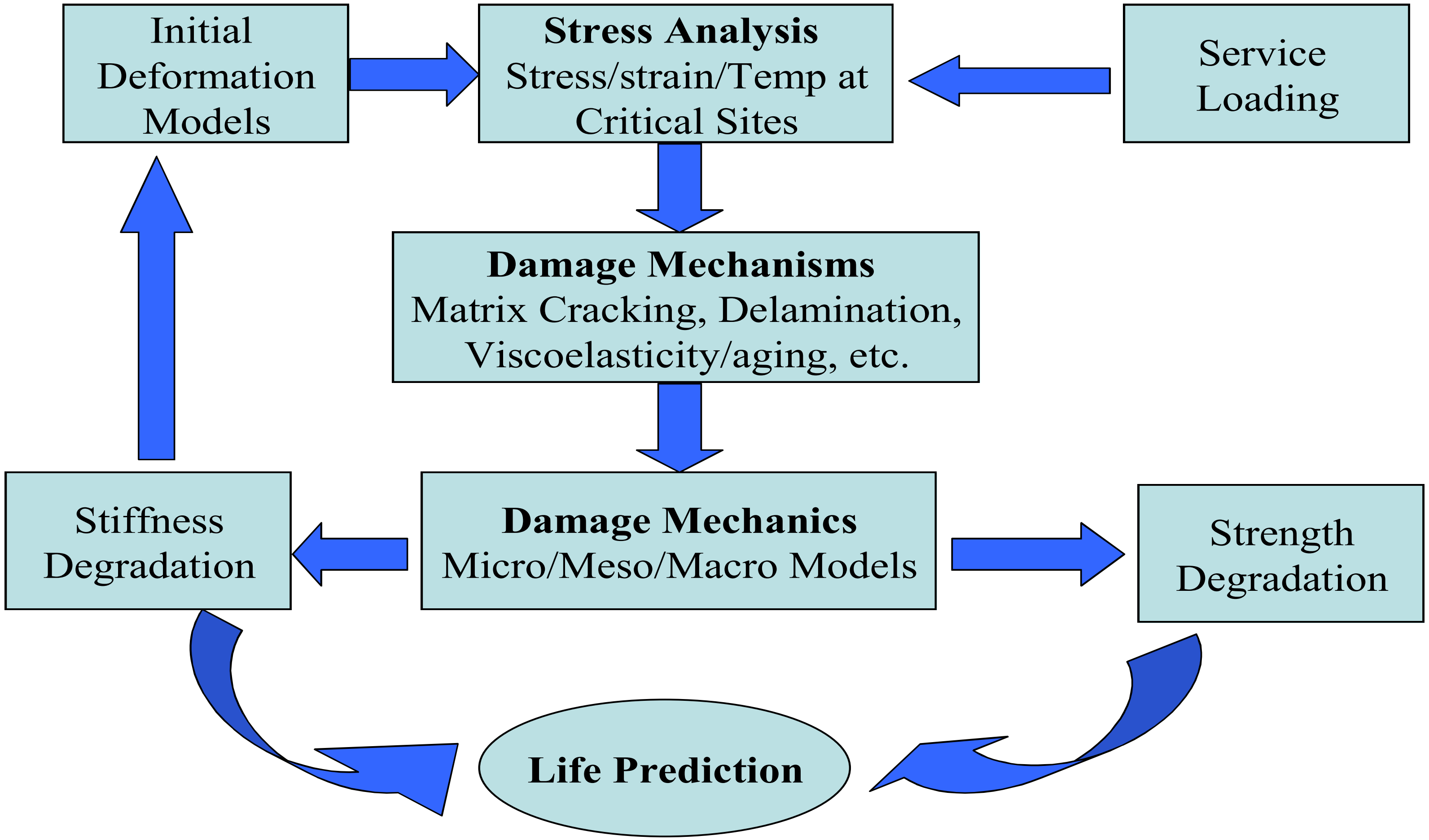
$$\frac{b_i}{b_r} = \frac{\sigma_{x,\text{ult}}(\lambda_C, \lambda_T, \theta)}{\sigma_{x,\text{ult}}(\lambda_{C,r}, \lambda_{T,r}, \theta_r)}.$$

$$g(R) = 1 \quad \text{if } R = R_r \text{ or } R \leqslant 0$$
$$g(R) = \frac{1 - R}{1 - R_r} \quad \text{if } R > 0,$$

Prediction vs Experimental data Using the Fewaz-Ellyin Criterion (local coordinates)



Overview of fatigue life prediction process



Way forward for a reliable fatigue life prediction